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PROBABILISTIC ANALYSIS OF WASTEWATER TREATMENT AND DISPOSAL SYSTEMS

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ABSTRACT

PROBABILISTIC ANALYSIS OF WASTEWATER TREATMENT AND DISPOSAL SYSTEMS

This work attempts to predict dissolved oxygen deficits in a stream with known initial conditions by taking into account the variations in deoxygenation and reaeration coefficients. A hypothetical stream situation is used to establish the significance in predicting dissolved oxygen deficit. Statistical models are formulated and tested for the variations in these coefficients using published data. Simulation techniques using the Monte Carlo method are employed in predicting the probabilistic variation in dissolved oxygen deficits for known initial conditions and the results are verified with the survey data observed for the Ohio River-Cincinnati Pool reach. The predicted results using probabilistic model are found to agree with the observed values within practical limits and give more consistent results than conventional methods.

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Though the standards adopted for dissolved oxygen are definitive in nature, there is a finite probability that the dissolved oxygen is likely to fall below the set standard during 16 hours of any 24-hour period and the standard set for the minimum dissolved oxygen at any time. There is a growing realization among the investigators for the need of probabilistic stream standards (Loucks, 1967; Ledbetter and Gloyna, 1964; Thayer, 1966). Loucks (1967) proposes a probabilistic stream standard for dissolved oxygen as follows:

> The dissolved oxygen concentration in the stream during any 7 consecutive day period must be such that
>
> 1. The probability of its being less than 4 mg/l for any 1 day is less than 0.2; and
> 2. The probability of its being less than 2 mg/l for any 1 day is less than 0.1 and for any 2 or more days is less than 0.05.

OBJECTIVE OF THE STUDY

In order to design any type of treatment plant, it is necessary to know what volume and concentration of pollutant may be discharged to the stream so that the stream standards are not violated. This implies that there should be methods to predict the response of the receiving stream to waste loads placed in it. Several mathematical formulations are available for predicting the dissolved oxygen responses, most of which are based on the pioneering work of Streeter and Phelps (1925). In all these formulations, the reaction velocity coefficients affecting the rates of BOD removal, atmospheric reaeration, etc., are taken as constants, though it was recognized by a few investigators (LeBosquet and Tsivoglou, 1950; Eckenfelder and O'Connor, 1961) that they are far from being constants. The main aim of this work is to ascertain the significance of the variations in the velocity coefficients K_1 and K_2

I. INTRODUCTION

The rapid expansion of population and industrialization has resulted in increasingly difficult problems of water resources management. The most critical of these problems is the protection of water resources from the ravages of pollution by the discharge of wastes which are increasing in both volume and complexity. There is a general awakening and a demand that something must be done about stream pollution. The daily news media report various aspects of water pollution with considerable frequency and in the past decade many national laws have been enacted dealing with Federal participation in pollution control activities.

If a pollution control program is to be both effective and economical, engineers must possess the ability to predict the effects of specific waste discharges on the environment. Without this knowledge, administrative determination of the required degree of treatment for waste discharges, existing or proposed, can only be speculative. A low estimate of the required degree of treatment will result in undesirable conditions, while an overestimate will create unjustifiable economic burdens on the waste discharger without commensurate benefit to the environment.

The determination of acceptable levels of water quality is usually the task of a regulatory agency. These governmental agencies have used several approaches to control the water quality in streams. One approach is through the use of treatment standards which require specified degrees of treatment, usually in terms of biochemical oxygen demand (BOD) reduction, suspended solids and coliform removal. Effluent standards have also been established which limit the concentrations of the various constituents in the effluent released into the stream. These regulatory activities are attuned to the concept of equity, which require the same degree of removal of BOD and solids from all waste sources irrespective of the volume of waste (Jacobs, 1965). These standards are used because they are relatively easier to administer, but each one deals only indirectly with the basic problem of stream quality.

The establishment of stream standards requires the polluter to regulate his effluent such that at least a minimum level of stream quality is maintained at all times. Though the aim of all the three types of standards is to assure acceptable levels of stream quality, it is directly accomplished only in the case of stream standards. The adoption of stream standards as a tool for regulatory control of pollution is not without limitations. In the first place it is very difficult to administer and enforce, especially if there is more than one polluter. Also, the application of rigid stream standards to large areas may well become a barrier to orderly economic development, thus defeating some of the benefits to be derived from the equitable use of water resources within a given locality or region (Jacobs, 1965).

The important point which needs to be made here is that the concept of stream standards and effluent standards is not mutually exclusive. In most cases both are necessary. For example, the Illinois Sanitary Water Board (1966) has adopted rules and regulations pertaining to sewage and industrial waste treatment requirements, effluent criteria, and water quality criteria for lakes and rivers, in order to protect the water resources of the state. It is true that the amount of weight placed on a particular type of standard depends on a given situation, the type of waste and the stage of development of the pollution abatement program. As pollution abatement progress is made and as the demands for various water uses grow, more and more emphasis will have to be given to the stream standards.

Though the stream quality standards specify minimum acceptable levels for a variety of stream quality parameters like bacteria, dissolved solids, chemical constituents, etc., the parameter most commonly used as a measure of the pollution from biodegradable waste, by investigators concerned with stream sanitation aspects, is the stream's dissolved oxygen (DO) concentration. In this study, the primary attention will be directed to this parameter, though it is conceptually feasible to extend the ideas to deal with other parameters as well.

With the enactment of the Water Quality Act of 1965, state regulatory agencies either have adopted or are in the process of adopting (as of May 1968) standards for interstate waters. Among other parameters considered for stream standards, the Illinois Sanitary Water Board (1966) has adopted different standards for dissolved oxygen in Illinois rivers, depending on the water use. Thus, the criteria adopted for the aquatic life sector of streams is:

> For maintenance of well balanced fish habitats the dissolved oxygen content shall be not less than 5.0 mg/l during at least 16 hours of any 24 hour period, nor less than 3.0 mg/l at any time,

and for the industrial water use sector, it is:

> Not less than 3.0 mg/l during at least 16 hours of any 24 hour period, nor less than 2.0 mg/l at any time.

Though the standards adopted for dissolved oxygen are definitive in nature, there is a finite probability that the dissolved oxygen is likely to fall below the set standard during 16 hours of any 24-hour period and the standard set for the minimum dissolved oxygen at any time. There is a growing realization among the investigators for the need of probabilistic stream standards (Loucks, 1967; Ledbetter and Gloyna, 1964; Thayer, 1966). Loucks (1967) proposes a probabilistic stream standard for dissolved oxygen as follows:

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> 2. The probability of its being less than 2 mg/l for any 1 day is less than 0.1 and for any 2 or more days is less than 0.05.

OBJECTIVE OF THE STUDY

In order to design any type of treatment plant, it is necessary to know what volume and concentration of pollutant may be discharged to the stream so that the stream standards are not violated. This implies that there should be methods to predict the response of the receiving stream to waste loads placed in it. Several mathematical formulations are available for predicting the dissolved oxygen responses, most of which are based on the pioneering work of Streeter and Phelps (1925). In all these formulations, the reaction velocity coefficients affecting the rates of BOD removal, atmospheric reaeration, etc., are taken as constants, though it was recognized by a few investigators (LeBosquet and Tsivoglou, 1950; Eckenfelder and O'Connor, 1961) that they are far from being constants. The main aim of this work is to ascertain the significance of the variations in the velocity coefficients K_1 and K_2

in defining the dissolved oxygen (DO) response of a receiving stream and to develop a procedure for determining the DO taking the variability in these rate coefficients into consideration, if these variations are significant.

The specific objectives of this study are:

1. To examine the relative importance of the variations in the reaction velocity coefficients affecting the bacterial oxidation of the organic matter and the atmospheric reaeration of river water, in predicting the dissolved oxygen responses of the receiving stream.

2. To determine the nature of variations of these velocity coefficients and to formulate and test the hypotheses concerning their chance variations.

3. To develop a procedure for predicting the dissolved oxygen in a river downstream of a waste source by taking into account the variations in these velocity coefficients.

4. To enumerate quantitatively the chance variations in dissolved oxygen responses in streams in terms of probability measure.

The last of the four objectives mentioned above is extremely significant in the light of the guidelines established for water quality standards under the Water Quality Act of 1965, wherein it is stated that numerical values for quality characteristics and the measure of limiting values which will govern for purposes of the criteria should be defined. The increasing importance of this concept will be felt as and when the stream quality standards are implemented and enforced, particularly with the advent of growing use of automatic stream monitoring installations.

SCOPE OF THE STUDY

The present study is limited to the following:

1. The relative importance of the variations in reaction velocity coefficients on the dissolved oxygen response in the stream are evaluated using a hypothetical stream situation based on Streeter-Phelps' formulation.

2. The Ohio River-Cincinnati Pool survey data for the river reach between miles 474.6 and 479.05, published by the U. S. Department of Health, Education and Welfare (1960) are used for determining the values of the velocity coefficients affecting deoxygenation rates in river samples.

3. The data collected by the Tennessee Valley Authority (1962) for the prediction of stream reaeration rates are used for formulating the hypothesis concerning the chance variation of the velocity coefficient governing the rate of atmospheric reaeration in rivers.

4. Statistical models describing the chance variations of the two velocity coefficients mentioned above are formulated and tested.

5. Using Monte Carlo techniques, two sets of data for the velocity coefficients are generated on the basis of the statistical models formulated. The generated data are tested and verified.

6. The generated data are used in the conceptual model for the dissolved oxygen-biological oxygen demand (DO-BOD) relationship in streams for predicting the DO response. The conceptual model is verified using the Ohio River-Cincinnati Pool survey data and the probabilistic variation of the DO responses are studied.

II. LITERATURE REVIEW

The philosophy of protecting the stream for most uses, even future uses, is being increasingly accepted, since it is good from the standpoint of the water user and the public. It permits planned growth of water-using industries and recreation side by side, and it prevents undue time lags between the need for water use and its realization. In recent years, there has been a great deal of attention towards the systems analysis approach to water quality management considering the river flow together with the possibilities of affecting or controlling the quality of the water at various use points by dams, water purification plants, and waste water and industrial waste treatment plants (Thomas and Burden, 1963; Liebman, 1965; Loucks, 1965; Montgomery, 1964; Worley, 1963). In most of these studies (Loucks, 1965; Liebman, 1965; Montgomery, 1964; Worley, 1963), the critical dissolved oxygen resulting from the addition of organic waste to a water course is taken as a controlling parameter for studying the system performance, where critical dissolved oxygen concentration is defined as the minimum concentration of dissolved oxygen in the river below a waste outfall. If such an approach is to yield more reliable information, it is imperative that the DO response of the receiving stream for the waste loads placed in it must be predicted with a greater degree of accuracy than is possible with the methods adopted in these studies.

The wastes discharged into a stream from municipal and industrial treatment plants contain a large variety of chemical compounds. Of primary interest in this work is that portion of the waste which is biodegradable and hence oxygen consuming. When this material is introduced in a water course, it undergoes biochemical oxidation, caused by microorganisms which utilize the organic matter for energy and growth.

If sufficient dissolved oxygen is present in the water, the microflora are primarily aerobic, utilizing the dissolved oxygen to carry out oxidation reactions producing water and carbon dioxide as end products. If, however, sufficient oxygen is not present, anaerobic organisms predominate resulting in undesirable end products. Also the concentration levels of oxygen have profound effects on the physiology and type of fishes found in the water bodies. The lethal effect of low concentrations of DO appears to be increased by the presence of toxic substances such as excessive dissolved carbon dioxide, ammonia, cyanides, zinc, lead or copper.

In situations where maintenance of well balanced fish habitat is one of the primary objectives, the stipulated standards for dissolved oxygen are that it shall not be less than 5.0 mg/l during at least 16 hours of any 24-hour period, nor less than 3.0 mg/l at any time (Illinois Sanitary Water Board, 1966; Aquatic Life Advisory Committee of ORSANCO, 1960). Where other uses like the industrial water are more important, the stream standards adopted for such uses with respect to DO differ from the above.

Because of the variety of oxygen-demanding organics in the wastes, it is common to measure the strength of wastes in terms of their biochemical oxygen demand (BOD) rather than to analyze for the chemical constituents of the wastes.

The most widely used mathematical models for predicting the DO responses in a stream are either the one proposed by Streeter and Phelps (1925) or the modified predictor equations based on Streeter-Phelps' formulation (Camp, 1963; Dobbins, 1964), though a few other concepts in DO predictions (Churchill and Buckingham, 1956; Thayer and Krutchkoff, 1965) have been put forth and applied with varying degrees of success.

The Streeter-Phelps equation considers only two mechanisms affecting DO, namely the removal of oxygen by bacterial oxidation of organic matter and the absorption of oxygen from the atmosphere. First order kinetics are employed to express deoxygenation and reoxygenation. The rate of removal of BOD on the basis of first order kinetics is given by:

$$\frac{dL}{dt} = -K_1 L \tag{1}$$

where L is the ultimate first stage BOD (milligram per liter) remaining to be satisfied at any time t (days) and K_1 (base e) is the reaction rate coefficient, which depends on the characteristics of the waste, temperature, type of microorganisms present, and other environmental factors. The integration of this equation yields

$$L = L_a e^{-K_1 t} \tag{2}$$

where L_a is the initial (t = 0) ultimate first stage demand. Since BOD is measured in terms of oxygen consumed, the rate of oxygen consumption equals the rate at which BOD is satisfied, i.e., $\frac{dD}{dt} = -\frac{dL}{dt} = K_1 L$, where

D is the dissolved oxygen deficit defined as the difference between saturation concentration, C_s, at the river water temperature and the oxygen concentration, C, in the river.

Reoxygenation process in the stream due to atmospheric re-aeration is also considered as a first order reaction which depends on the dissolved oxygen deficit, D. This can be expressed mathematically as:

$$\frac{dC}{dt} = K_2 (C_s - C) = K_2 D = -\frac{dD}{dt} \qquad (3)$$

where K_2 (base e) is the reaction rate coefficient defining the re-aeration process.

Combining the rates of these two reactions, and writing the resulting equation in terms of dissolved oxygen deficit, the response of the receiving stream for a single waste source is defined by the differential equation as:

$$\frac{dD}{dt} = K_1 L - K_2 D \qquad (4)$$

The solution of this differential equation with appropriate initial conditions yields the well known and classical Streeter-Phelps equation for stream self purification capacity and is given as:

$$D = \frac{K_1 L_a}{K_2 - K_1} \left[\exp(-K_1 t) - \exp(-K_2 t)\right] + D_a \exp(-K_2 t) \qquad (5)$$

where D_a is the initial dissolved oxygen deficit in the river reach at time $t = 0$ and all other terms are as defined previously.

Figure 1 shows the typical oxygen sag curve. The point of maximum deficit (or minimum dissolved oxygen concentration) is known as the critical point. The critical time, or time at which the maximum deficit occurs (t_c), and the corresponding maximum or critical deficit (D_c) are defined by the following equations as:

$$t_c = \frac{1}{K_2 - K_1} \{\ln [\frac{K_2}{K_1} (1 - \frac{(K_2 - K_1)}{K_1 L_a} D_a)]\} \qquad (6)$$

and

$$D_c = \frac{K_1}{K_2} L_a \exp (-K_1 t_c) \qquad (7)$$

In the river reach prior to the critical point, the rate of deoxygenation is greater than the reaeration rate and beyond this point, the converse holds. The river is said to recover in the latter region from the pollutional load discharged into it.

The shortcomings and criticisms expressed against the Streeter-Phelps equation, Eq. 5, for stream self purification capacities and attempts of other investigators to modify these concepts in order to improve upon the predictions are discussed in the following paragraphs. There are also altogether different approaches taken for the prediction of DO response of the streams to waste loads placed in it and these are discussed along with the suggested modifications for the Streeter-Phelps concepts.

The Streeter-Phelps sag equation when fitted to stream DO and BOD data have yielded such a wide range of values of the parameters K_1 and K_2 as to suggest that some important factors have not been taken

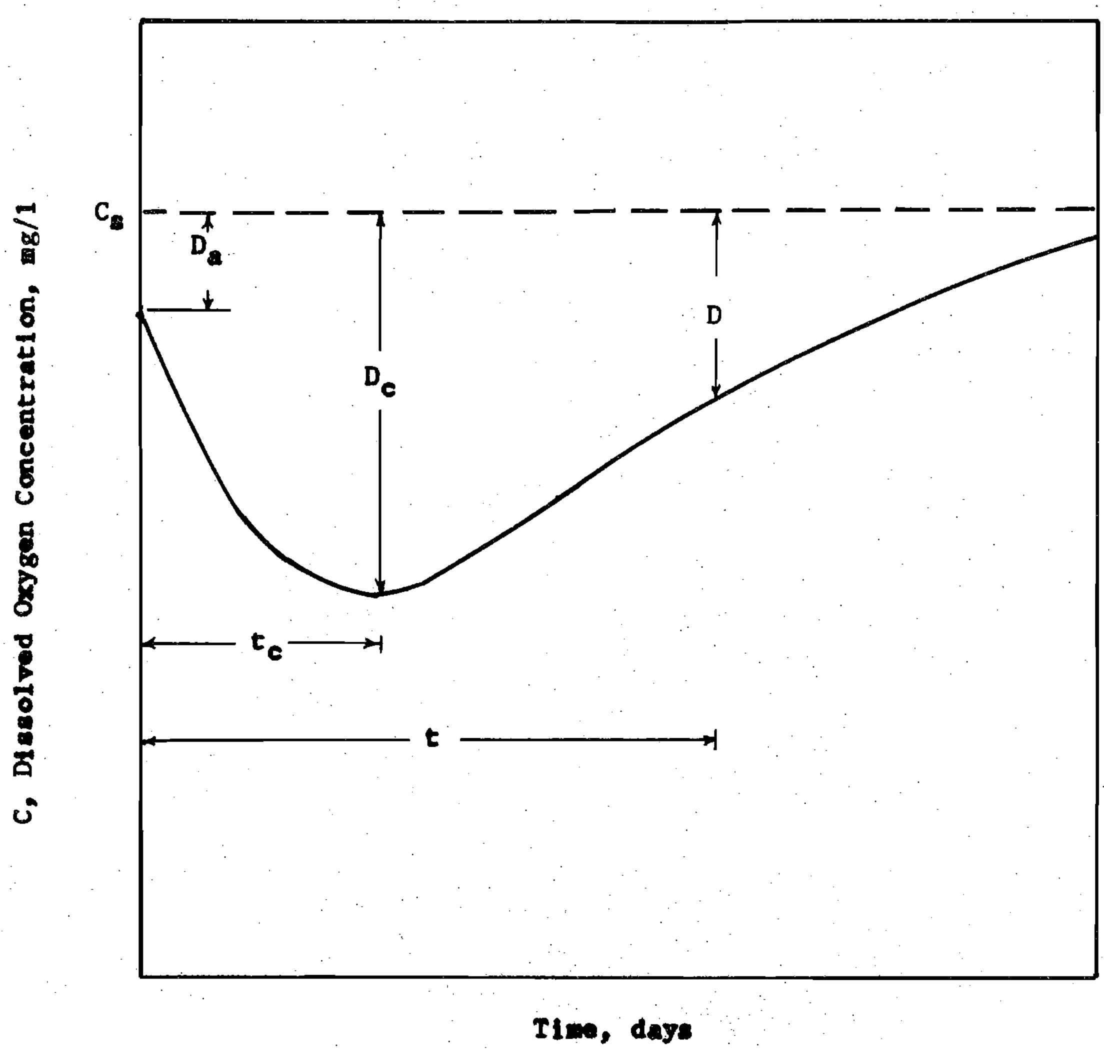

FIGURE 1. DISSOLVED OXYGEN SAG CURVE

into account in the formulation or that the variables have not been incorporated in the correct mathematical form. Several authors (Camp, 1963; Dobbins, 1964; O'Connor, 1967) have proposed modifications to the Streeter-Phelps formulation by taking into account other parameters which affect the oxygen uptake in the stream like removal of BOD due to adsorption and sedimentation, addition of BOD from benthal layers to the overlying water, benthal oxygen demand, oxygen addition due to photosynthesis, algal respiration, etc.

Some apprehension also exists concerning the validity of the application of the first order concept and how the results of the BOD test should be interpreted mathematically (Gates, 1966; Thomas, 1961; Young, 1965). Thomas (1961) assumes second order (bimolecular) reaction kinetics for BOD removal, in his "Step Method" for estimating the oxygen uptake in a stream. The advantage claimed by the author is that the variation of the BOD reaction velocity parameter is considerably reduced if a second order rate equation is used in place of the first order equation. Young (1965) concludes, based on the analysis of the data for sewage, that the first and second order kinetics fit the first stage BOD data with equal precision at 20°C and 35°C. Since the validity of the second order kinetics concept over the first order kinetics has not been well established, most of the recent works (Liebman, 1965; Loucks, 1965; O'Connor, 1967; Thayer, 1966) use first order kinetics for the oxygen depletion due to bacterial degradation of organic matter. Inasmuch as this concept is more widely recognized and used in the sanitary engineering field, the BOD progression will be assumed to follow first order kinetics.

The merit of employing a mathematical model developed from data obtained in such a nondynamic system as BOD bottle and applying it directly to a dynamic system such as a stream has been questioned in the recent past (Gates, 1966; Gannon, 1963; Nejedly, 1966). Gannon (1963) states

> ...it is apparent that no adequate explanation is available to account for the differences between the laboratory k_1 and the river k_r. This, no doubt, is partially due to inadequacies of the existing BOD techniques. Probably it may never be possible to develop a BOD procedure in the laboratory that will duplicate the natural river conditions, but certainly efforts should be directed toward improving existing procedures.
>
> In addition to improvement of the laboratory BOD test it appears that much useful fundamental information could be obtained about BOD removal rates from controlled studies using artificial channels. This approach, then, would be somewhere between the natural stream conditions and the highly artificial bottle methods.

Isaacs and Gaudy (1967) from their studies specifically designed to gain an insight into the equivalence of K_1 determined in the bottle and in a laboratory stream model, using synthetic sewage consisting of equal parts of glucose and glutamic acid came to the conclusion that data obtained in BOD bottles could be used to make a reasonable prediction of the critical DO provided the seed concentration in the BOD bottle was identical, or nearly identical, to the biological solids concentration in the receiving stream. They further observe, based on the model stream study, that a fairly accurate prediction could be made by estimating the extent of the first stage exertion. Since these conditions could be easily met by making observations on river samples obtained downstream of a waste source, the idea of extrapolating the BOD bottle data to river situations cannot be ruled out, at least until a better method could be devised and established.

Nejedly (1966) holds the view that longitudinal mixing, i.e., mixing in the direction of flow, induces contacts between particles of decaying organic substances with different detention periods within the river stretch considered, and hence a more rapid BOD reduction takes place in a stream than in a bottle. This view is opposed to those of Camp (1965), Dobbins (1964), O'Connor (1967) and several others who hold that the effect of longitudinal dispersion is negligible in determining streams' assimilative capacity. Though, Nejedly presents data to support his hypothesis, no quantitative conclusion for the effect of longitudinal mixing on the reaction coefficient K_1 could be determined.

It is generally recognized that the differences in removal rates of BOD in BOD bottles and in actual streams are due to the phenomena of biological accumulation and removal of BOD due to sedimentation without the DO being affected (Gannon, 1963; O'Connor, 1967). These two removal rates are taken as the same in the absence of settleable organic solids and fixed aquatic vegetations in the receiving stream (O'Connor, 1967).

In spite of the fact that the bottle BOD technique has been subjected to severe criticism, it has been used in one form or the other in determining stream assimilative capacities. Recognizing that deoxygenation due to bacterial degradation of organic matter is only one of the several mechanisms which affect the BOD removal in a stream, there seems to be ample justification in using the data obtained in BOD bottles to determine river self purification capacities, particularly when the mechanisms affecting the BOD removal without oxygen consumption are considered separately in mathematical modeling.

Another severe criticism voiced against the use of Streeter-Phelps equation or its modifications is that the so-called reaction velocity constants K_1, K_2, etc., are found to vary considerably. LeBosquet and Tsivoglou (1950) observe:

> The velocity constant for reoxygenation, K_2, is subject to wide variation in different streams, in individual stretches of any one stream and for various river stages in the same stretch. Although variations in K_1 are usually of somewhat lesser magnitude, they too may be appreciable, and in many situations it is quite difficult, if not impossible, to derive acceptable values for these velocity coefficients.

Churchill and Buckingham (1956) state regarding the variability of these coefficients as follows:

> The so called constants K_1 and K_2 are determined from an intensive survey, usually at a fairly low, steady, flow. However, if another survey is made on the same reach, even fairly soon after the first one, it at once becomes obvious that the "constants" may vary <u>considerably</u> from the values first determined....Therefore, when the derived "constants" are far from constant, the results of computation involving these "constants" for other conditions of load, flow, and temperature, are certainly open to question.

The main aim of this work as stated earlier is to ascertain the significance of the variations in the velocity coefficients K_1 and K_2 in defining the DO response of a receiving stream and to develop a procedure for determining the DO taking the variability in these rate coefficients into consideration, if these variations are significant.

Velz (1939) presented a method for oxygen balance studies in polluted rivers in which the atmospheric river reaeration is computed from an empirical relationship, established in a quiescent column of water, defining the relationship of the terms: percent saturation after a known elapsed time, linear depth of column, molecular diffusion coefficient, and the initial percent saturation of the column of water. The application of an empirical relationship established in a quiescent body of water to a dynamic system as in a river situation is highly questionable, particularly when the effect of molecular diffusion of oxygen is considered negligible in a stream. Also a family of curves, giving the relationship between BOD and time at temperatures 0°C to 30°C are presented for the purpose of determining the BOD reduction along the course of the river. Though first order reaction kinetics is assumed in defining these curves, no distinction is made for the differences in the characteristics of the wastes under examination, i.e., the percent reduction of the pollutional load in a given flow-through time, is taken as independent of the nature of the waste. Velz's method suffers from the same shortcomings discussed earlier for the Streeter-Phelps formulation and in addition this method does not take cognizance of the nature of wastes which is an important factor in biodegradability.

Thomas (1948) discussed a technique for stream analysis which recognizes that the K_1 value may not be 0.23, the commonly accepted value in sanitary engineering practice, and also that there may be BOD sinks other than that represented by bio-oxidation. Furthermore, his discussion indicates that he did not consider these additional BOD sinks to be oxygen sinks. This technique requires that, through the use of the BOD test, the values of K_1 and ultimate first stage demands be determined at boundary stations of a hydraulically uniform stretch of stream. The difference between the ultimate first stage BOD values of the upstream and downstream stations represents the reduction in oxygen demand realized through the reach. This information combined with the time of passage, and assuming first order kinetics, is used to determine the rate constant expressing the reduction in potential oxygen demand by the biomass is determined by averaging the K_1 values observed for samples from the two boundary stations. The difference between this rate and the rate observed for the stream represents the rate at which potential oxygen demanding material is going to non-oxygen consuming sinks.

Although this approach recognizes that K_1 may vary, it assumes that the stream values of K_1 and L can be predicted from bottle measurements. The concept that not all potential oxygen consumption need necessarily be satisfied by actual consumption was a significant contribution to stream analysis. But for this modification, Thomas' technique is identical to that employed by Streeter and Phelps (1925) and subject to the same previously-indicated reservations.

LeBosquet and Tsivoglou (1950) published data for the Ohio River indicating a radically different method of pollutional analysis employing statistical methods. They established a linear relationship between critical DO deficit in mg/l and reciprocal of river flow in cubic feet per second, with a high degree of correlation. This method permits direct analysis of oxygen-flow relationship in a given stretch avoiding the use of factors such as K_1 and K_2. The method outlined by them cannot give better results than the oxygen sag formula since the characteristics of the wastes and the reoxygenation capacities of the stream fluctuate greatly and such variations will introduce inaccuracies. It is based mainly on the surmise that the effect of the variations in K_1 and K_2 are relatively minor, which is not a valid conjecture as will be borne out by this study.

Churchill and Buckingham (1956) extended the concept of LeBosquet and Tsivoglou (1950) and presented an analysis employing statistical techniques of multiple correlation and giving the maximum deficit in terms of a linear equation with 5-day BOD, water temperature and stream discharge as variables. The advantages claimed for this method are that evaluation of factors like K_1, K_2, L and flow-through time are avoided and that the correlation analysis predicts the critical value reasonably well.

The authors (Churchill and Buckingham, 1956) recommend choosing a station for making 5-day BOD observations downstream of the pollution, and as close to the critical section as possible. Since the critical section in the river fluctuates with variations in stream flow, and the characteristics of the waste, the criteria suggested by the authors for selection of site for making BOD observations to be used in the regression analysis is not easily met since it is not feasible to estimate the location of the critical section from their statistical model. Also, it is not possible to predict the state of oxygen concentration in different sections of the river using the correlation technique. It is of paramount importance to estimate assimilative capacity at any desired reach and not merely the critical reach of the river, particularly when comprehensive river basins planning are envisaged with planned development of rivers.

Thomas (1961) presented a method called the "Step Method" for estimating oxygen uptake in a stream. Like the Streeter-Phelps formulation, only two mechanisms affecting DO in a stream are considered, namely the removal of oxygen due to bacterial decomposition and addition of oxygen from the atmosphere, even though he has emphasized earlier (1948) the importance of considering removal and addition of BOD due to sedimentation and scour respectively. The only departures from the classical Streeter-Phelps formulation are that the bacterial degradation of organic matter is assumed to be governed by second order kinetics and that the values of 5-day BOD are used instead of the ultimate first stage BOD. In the light of the investigations of Young and Clark (1965), and Isaacs and Gaudy (1967) discussed earlier, the "Step Method" does not appear to improve the predictive techniques for stream self purification capacities.

Camp (1963), and Dobbins (1964) proposed equations which are very similar, for predicting dissolved oxygen deficit by taking into account benthal demand, removal or addition of BOD due to deposition or scour, and photosynthesis. O'Connor (1967) further extended these ideas by considering the diurnal variation of oxygen addition due to photosynthesis instead of treating it as time constant for the whole 24-hour period as in Camp's model, and adding another term for algal respiration in the oxygen sinks. Since these formulations (Camp, 1963; Dobbins, 1964; O'Connor, 1967) are only modifications of the basic Streeter-Phelps formulation, considering a few more plausible mechanisms affecting the DO-BOD relationship in streams, they do not obviate all the criticisms put forth for the Streeter-Phelps equation.

Thomann (1963) postulated a mathematical model using the systems analysis concept for predicting the response of any water quality parameter, affected either by conservative or nonconservative pollutants. The model is developed for a one-dimensional estuary for which stream analysis is only a particular case. The model assumes that a body of water can be segmented into a discrete number of parts each of which is treated as a stationary linear subsystem forming part of the whole system. The time rate of change of a water quality parameter particularly DO, in any of the subsystems is taken as affected by the advection and diffusion of DO both into and out of the system and by the sources and sinks of DO within each subsystem. These additions or removals of DO are considered as the variables that force the DO to respond. An equation representing statinary linear subsystem for DO concentration at a given time, with the sources and sinks of DO as forcing functions and also with feed-back mechanisms from adjoining subsystems, is developed. Since the subsystems that form the overall system are all linear, the solutions obtained for each subsystem are linearly superimposed to predict the response of the system as a whole.

Though Thomann (1963) has indicated conceptually a method for predicting the DO response when the decay coefficient for organic matter changes with time, no mention is made in his theoretical development, how the variations with time in the decay coefficients are evaluated. Moreover, the possible variations in the reaeration coefficient are overlooked. It is well documented (TVA, 1962) that the maximum reaeration coefficient observed under constant flow conditions in the Tennessee Valley rivers, when the temperatures remained nearly uniform, ranged approximately from 2 to 9 times the minimum values depending on the characteristics of the river. Thomann's work marks a significant advance in pollution analysis wherein an attempt has been made to consider all the factors affecting DO in a body of water taking into account the possible variations in some of the parameters affecting the system.

Thayer (1966) viewed the oxygen relationship in streams as a stochastic birth and death process with the BOD and DO being increased and decreased by small amounts in a very short interval of time. The stochastic model provides for the joint density function for both pollution and dissolved oxygen for different initial conditions. The advantage claimed by the author is that in addition to predicting the mean DO concentration which is the same as that predicted by the deterministic equation of Dobbins (1964), it affords a measure of the variance of DO from its mean value, thus enabling one to predict the probability of DO downstream of a source of pollution taking any given value.

In developing his model, Thayer (1966) did not consider the effects of photosynthesis and algal respiration. The prediction equation was verified with the data published by the Resources Agency of California (1962) for the Sacramento River. Though the published data indicate a value for K_1 ranging from 0.12 to 0.55 and emphasize the importance of photosynthesis and algal respiration in the river reach below Sacramento, these effects were totally ignored in the model verification. Also in the controlled laboratory experiments with dextrose as the only substrate for simulating river conditions, values ranging from 0.076 to 0.167 were obtained for K_1 (Thayer, 1966). Whereas, in the verification of his theory with the results of the laboratory data, a value of 0.07 for K_1 has been assumed on the supposition that the assumed value is reasonably close to the observed values. In the opinion of this writer, the verification of the mathematical model (Thayer, 1966) either with the published data for the Sacramento River or with the laboratory data leave much to be desired.

From the foregoing general discussion on the present state of knowledge concerning the stream assimilative capacities, it can be summarized that though there are strong indications from field measurements for the variability in the parameters affecting DO-BOD relationship, and though a need for taking into account these variations is strongly felt, all the theories so far postulated except that of Thomann (1963) fail to consider this aspect. Even though there is considerable criticism for extrapolating the BOD bottle observations to river conditions, this is the only expedient way available at the present time for defining the biological interactions in the environment. The concept of first order kinetics, first introduced by Streeter and Phelps (1925), with the parameters determined by bottle BOD tests will continue to be used until a better and more reliable method is evolved to substitute this concept.

III. SENSITIVITY ANALYSIS

DEFINITION

Sensitivity analysis applies to the concept in which all except one of the variables in a system are held constant and this single exception is allowed to vary through its full possible range. The effects on certain system performance are noted, testing the sensitivity of the system to that particular variable which is varied. If the measured output varies significantly, the system is said to be sensitive to that variable for the given conditions. In a mathematical sense, the slope of the output as a function of input is a sensitivity measure.

Two variables of a system may be permitted to vary simultaneously, each having an effect on the output measure. Mathematically this becomes a three-dimensional model; the output function becomes a surface rather than a line. This is called a response surface. A well defined response surface contains outcomes of all possible combinations of the two variables. In a mathematical sense, the sensitivity of the response surface to a single independent variable is a partial derivative of the response surface equation with respect to that variable.

PROCEDURE

The suggested general procedure for carrying out a sensitivity analysis is as follows:

1. The independent variables of the system are listed and the possible range of values for each of the variables is ascertained.

2. The value of the response surface variable is calculated using nominal values. This value serves as a reference point.

3. Percent change in the response surface is computed by substituting the low value of one of the independent variables.

4. The low value is replaced with an incremented value for the same variable as in step 3 and the percent change in the response surface is evaluated. This is repeated until all the values within the possible range of values for the variable under consideration are covered.

5. Steps 3 and 4 are repeated for each of the independent variables which is likely to assume a range of values instead of a single value.

The above procedure for individual variables analysis can be modified to test for changes of variables in groups which may be related.

Nominal Values in the Streeter-Phelps Equation

The Streeter-Phelps equation for stream assimilative capacity (Eq. 5) will be used in the sensitivity analysis for determining the significance of the variations in reaction velocity coefficients K_1 and K_2 in predicting the dissolved oxygen response. It is generally accepted in sanitary engineering practice that the average value for K_1 at 20°C for domestic wastes is 0.23 per day (base e) (Fair and Geyer, 1954; Sawyer, 1960), even though values ranging from 0.1 to 0.7 per day are indicated by some authors for domestic and industrial wastes (Camp, 1963; Eckenfelder and O'Connor, 1961). The K_1 value at 20°C reported for river samples collected downstream of Sacramento sewage treatment plant discharge is found to range from 0.12 to 0.55 per day with an average value of 0.294 per day (The Resources Agency of California, 1962). The computed K_1 values for the Ohio River samples obtained during a 1957 survey (USPHS, 1960) in the river reach between miles 474.6 and 479.05, where all the pollution entered prior to the reach under consideration, showed a range from 0.05 to 0.32 per day with an average value of 0.173 per day. From the average value cited for K_1 in the literature and from the values of K_1 obtained in actual river surveys, it can be concluded that the nominal (average) value for the purpose of sensitivity analysis can be taken as 0.23 per day at 20°C.

The nominal value for K_2 at 20°C is not very well defined since it depends on the characteristics of the stream. The classification of streams from reaeration point of view is only subjective in nature. Fair and Geyer (1954) indicate for large streams a value of K_2 (base e) at 20°C ranging from 0.4 to 0.7 per day. Eckenfelder and O'Connor indicate a common range of K_2 from 0.2 to 10.0 per day, the lower value representing deep slow-moving rivers and the higher value for rapid shallow streams. In one set of experiments (No. 14) in the Holston River, which could be classified as a large river with moderate velocity, K_2 had a range of values from 0.10 to 1.18 per day with an arithmetic average of 0.63 per day under comparable conditions within practical limits (Buckingham, 1966). Similar ranges of values were obtained in a few other experiments conducted in TVA rivers. Hence an average value, in the case of large streams, for K_2 equal to 0.6 per day at 20°C will be a realistic assumption.

Temperature Effects on Reaction Coefficients

The importance of temperature effects on the velocity coefficients K_1 and K_2 has long been well recognized and much attention has been focused on this aspect. Streeter and Phelps (1925) described the relationship between K_1 and temperature by the expression

$$K_{1(T)} = K_{1(20)}\theta^{T-20} \qquad (8)$$

in which $K_{1(T)}$ is the deoxygenation coefficient at any temperature T in degrees centigrade, $K_{1(20)}$ is the deoxygenation coefficient at 20°C and θ is the temperature coefficient. θ was found to have an average value of 1.047. Theriault, as reported by Camp (1963), and Thomas (1961), confirmed this value of θ, based on his Ohio River studies. Recently, Zanoni (1967) in an attempt to establish the temperature effects on the rate of deoxygenation of a conventional activated sludge waste water treatment plant effluent, found the value of θ to be 1.048 for the temperature range of 10 to 30°C. Though the observed values for θ varied within a small range, the effects of this variability in estimating the deoxygenation rate at other temperatures, knowing its value at 20°C, will not be significant. This temperature relationship with θ having a value of 1.047 will be used in computing the nominal values at other temperatures, as has been employed in earlier works (Eckenfelder and O'Connor, 1961; O'Connor, 1960; Streeter and Phelps, 1925; Thomas, 1961; Worley, 1963).

Also, it has long been known from experimental evidence that deoxygenated water will absorb oxygen from the atmosphere at a higher rate if the temperature of the water is raised, other conditions being held constant. In the normal range of stream temperatures, a rise in water temperature results in a decrease in viscosity, density, and surface tension. It is difficult to distinguish the exact role played by each of these factors. Because it is only the net overall effect of temperature on reaeration that is of interest in most investigations, experiments have been conducted in the past for measuring reaeration rates at several temperature levels. Results of such experiments have been generally reported in the following form:

$$K_{2(T)} = K_{2(20)}\Phi^{T-20} \tag{9}$$

in which $K_{2(T)}$ is the reaeration coefficient at temperature T in degrees centigrade, $K_{2(20)}$ is the reaeration coefficient at 20°C and Φ is the temperature coefficient. Several authors indicated values for Φ ranging from 1.008 to 1.02 as reported by the Committee on Sanitary Engineering Research in their Thirty-first Progress Report (1961). The committee (1961) concluded, after conducting a series of carefully controlled experiments that the thermal coefficient Φ remains constant over a wide range of turbulence conditions, with a value of 1.0241. The relationship shown in Eq. 9 with Φ equal to 1.0241 will be used in computing the nominal values at other temperatures.

Table 1 shows the nominal values of the reaction velocity coefficients K_1 and K_2 at temperatures 10°C, 20°C, and 30°C used in the sensitivity analysis.

TABLE 1

NOMINAL VALUES OF REACTION COEFFICIENTS K_1 AND K_2

Parameters	Nominal Values		
	At 10°C	At 20°C	At 30°C
K_1	0.15	0.23	0.36
K_2	0.47	0.60	0.76

The range of values considered for K_1 and K_2 are 0.1 to 0.6 per day and 0.2 to 3.0 per day respectively with incremented values of 0.1 per day for K_1 and 0.2 per day for K_2. The initial conditions assumed for ultimate BOD and DO deficit in the hypothetical stream at the source of pollution are 20 mg/l and 1.5 mg/l respectively, which are considered as realistic values for these parameters, having been based on actual observations made on river samples (Gannon, 1963; USPHS, 1960). Since the effect of temperature on ultimate BOD is considered insignificant (Gotaas, 1948; Thomas, 1961; Zanoni, 1967), no correction for temperature effects will be made for estimating initial ultimate BOD at different temperatures considered in the sensitivity analysis.

RESULTS

The critical DO predicted by the Streeter-Phelps equation is considered for evaluating the response of the hypothetical stream system. The value obtained for critical DO, when K_1 and K_2 take their respective nominal values, is treated as the nominal value of critical DO for the given conditions of initial BOD, DO deficit, and temperature. Percent deviations of the system responses are computed when K_1 and K_2

take different values within the possible range and these are based on the reference value for the critical DO in the stream.

Eq. 5, which expresses the relationship between D, L_a, D_a, t, K_1 and K_2, becomes indeterminate when K_1 and K_2 are equal. Since the ranges of values, which these two coefficients can assume, overlap it is necessary to predict the critical deficit when K_1 equals K_2. The solution for the Streeter-Phelps formulation, Eq. 4, is given below for the case when K_1 and K_2 are equal.

$$D = K_1 L_a t \exp(-K_1 t) + D_a \exp(-K_1 t) \tag{10}$$

The time of flow for critical deficit, and critical deficit are:

$$t_c = \frac{L_a - D_a}{K_1 L_a} \tag{11}$$

$$D_c = L_a \exp(-K_1 t_c) \tag{12}$$

A computer program incorporating the general Streeter-Phelps equation, and the solution for the situation when K_1 equals K_2 was written in FASTRAN langaage for the IBM 7094-1401 system to evaluate the state of oxygen concentration at 1, 2, 3, 4, and 5 days of flow-through time in order to trace the oxygen sag profile. The program computes the flow-through time for critical deficit and hence the critical deficit and critical DO for different combinations of values for K_1 and K_2 with the assumed initial conditions for L_a and D_a. The flow diagram for the sensitivity analysis computer program is shown in Appendix B. Knowing the critical DO in the stream, it is then possible to reckon the percent

TABLE 3

EFFECT OF VARIATION IN K_2 ON PERCENT
ERROR IN PREDICTING CRITICAL DO AT 20°C

Saturation DO, mg/l	K_1, per Day	K_2, per Day	Critical Deficit, mg/l	Percent Saturation	Percent Deviation from Nominal Value
9.20	0.23	0.20	8.49	7.72	84.63
		0.40	5.88	36.09	28.14
		0.60	4.58	50.22	0.00*
		0.80	3.78	58.91	17.30
		1.00	3.23	64.89	29.21
		1.20	2.84	69.13	37.65
		1.40	2.53	72.50	44.36
		1.60	2.29	75.11	49.56
		1.80	2.10	77.17	53.66
		2.00	1.94	78.91	57.13
		2.20	1.81	80.33	59.96
		2.40	1.70	81.52	62.33
		2.60	1.61	82.50	64.28
		2.80	1.55	83.15	65.58
		3.00	1.50	83.70	66.67

*Nominal value.

than the assumed average value, the rate of reaeration will be considerable compared to the deoxygenation rate with the result that the critical deficit in the system will be the initial deficit itself. This upper bound for the percent error in DO prediction varies with the temperature.

Table 5 shows the sensitivity of the system for the variations in K_1 at 10°C, 20°C, and 30°C. These are plotted in Figure 3. Figures 4 to 6 show the relative effects of the deviations in K_1 and K_2 on the sensitivity of the system at temperatures 10°C, 20°C, and 30°C respectively. The curve marked as "lower scale" indicates the sensitivity of the system when K_2 varies and all other parameters of the system remain constant. Likewise the curve marked as "upper scale" refers to the sensitivity of the system when K_1 varies while other parameters remain constant. It is seen from these curves that except at 10°C, the variations in the velocity coefficients are equally significant in predicting the critical DO. At 10°C, the error in prediction when K_1 takes values greater than its nominal value is much more than the error due to the variations in K_2.

Generally, it is seen that the errors in prediction due to the variations in the reaction coefficients are considerable, with the result that there is no justification for using average values for these parameters in predicting dissolved oxygen concentrations. Hence it is postulated that by treating the velocity coefficients as variable coefficients instead of as constant coefficients, the values predicted by mathematical modeling can be made to approach the true value for DO

deviations in the predicted DO from the reference value when one of the reaction coefficients takes different values within its practical range of values while the other reaction coefficient assumes its nominal value and all other parameters are held at their respective constant values.

Tables 2 to 4 show the sensitivity of the hypothetical river system for the variations in K_2 at 10°C, 20°C, and 30°C respectively for the assumed initial conditions of BOD (20 mg/l) and DO deficit (1.5 mg/l). These are shown plotted in Figure 2. When K_2 takes the nominal value at a particular temperature, there will be no error in predicting the response of the system. However, when the actual value of K_2 is lesser or greater than the nominal value, the actual value for critical DO will be different from its nominal (reference) value. The figure clearly shows the temperature dependence of the sensitivity of the system for the variations in K_2. The error in prediction increases with increased deviations of the coefficient K_2 and this becomes increasingly significant at higher temperatures. Also it should be noted that there are upper bounds for these errors when the actual values for K_2 are lesser or greater than the nominal value. When the value is far less than the nominal value, the rate of deoxygenation exceeds the rate of reaeration with the result that the critical deficit could reach its limiting value equal to the DO saturation concentration for the given temperature. In such a case, maximum deviation could only be 100 percent, and this effect is seen for the situation when K_2 equals 0.2 per day at 30°C. On the other hand when K_2 has a value considerably greater

TABLE 3

EFFECT OF VARIATION IN K_2 ON PERCENT ERROR IN PREDICTING CRITICAL DO AT 20°C

Saturation DO, mg/l	K_1, per Day	K_2, per Day	Critical Deficit, mg/l	Percent Saturation	Percent Deviation from Nominal Value
9.20	0.23	0.20	8.49	7.72	84.63
		0.40	5.88	36.09	28.14
		0.60	4.58	50.22	0.00*
		0.80	3.78	58.91	17.30
		1.00	3.23	64.89	29.21
		1.20	2.84	69.13	37.65
		1.40	2.53	72.50	44.36
		1.60	2.29	75.11	49.56
		1.80	2.10	77.17	53.66
		2.00	1.94	78.91	57.13
		2.20	1.81	80.33	59.96
		2.40	1.70	81.52	62.33
		2.60	1.61	82.50	64.28
		2.80	1.55	83.15	65.58
		3.00	1.50	83.70	66.67

*Nominal value.

than the assumed average value, the rate of reaeration will be considerable compared to the deoxygenation rate with the result that the critical deficit in the system will be the initial deficit itself. This upper bound for the percent error in DO prediction varies with the temperature.

Table 5 shows the sensitivity of the system for the variations in K_1 at 10°C, 20°C, and 30°C. These are plotted in Figure 3. Figures 4 to 6 show the relative effects of the deviations in K_1 and K_2 on the sensitivity of the system at temperatures 10°C, 20°C, and 30°C respectively. The curve marked as "lower scale" indicates the sensitivity of the system when K_2 varies and all other parameters of the system remain constant. Likewise the curve marked as "upper scale" refers to the sensitivity of the system when K_1 varies while other parameters remain constant. It is seen from these curves that except at 10°C, the variations in the velocity coefficients are equally significant in predicting the critical DO. At 10°C, the error in prediction when K_1 takes values greater than its nominal value is much more than the error due to the variations in K_2.

Generally, it is seen that the errors in prediction due to the variations in the reaction coefficients are considerable, with the result that there is no justification for using average values for these parameters in predicting dissolved oxygen concentrations. Hence it is postulated that by treating the velocity coefficients as variable coefficients instead of as constant coefficients, the values predicted by mathematical modeling can be made to approach the true value for DO

TABLE 4

EFFECT OF VARIATION IN K_2 ON PERCENT ERROR IN PREDICTING CRITICAL DO AT 30°C

Saturation DO, mg/l	K_1, per Day	K_2, per Day	Critical Deficit, mg/l	Percent Saturation	Percent Deviation from Nominal Value
7.6	0.36	0.20	7.60	0.00	100.00
		0.40	7.52	1.05	96.63
		0.60	6.02	20.79	33.34
		0.76	5.23	31.39	0.00*
		0.80	5.07	33.29	6.73
		1.00	4.39	42.24	35.42
		1.20	3.89	48.82	56.52
		1.40	3.50	53.95	72.97
		1.60	3.18	58.16	86.47
		1.80	2.92	61.58	97.44
		2.00	2.71	64.34	106.28
		2.20	2.52	66.85	114.33
		2.40	2.37	68.82	120.65
		2.60	2.23	70.66	126.54
		2.80	2.11	72.24	131.61
		3.00	2.00	73.68	136.22

*Nominal value.

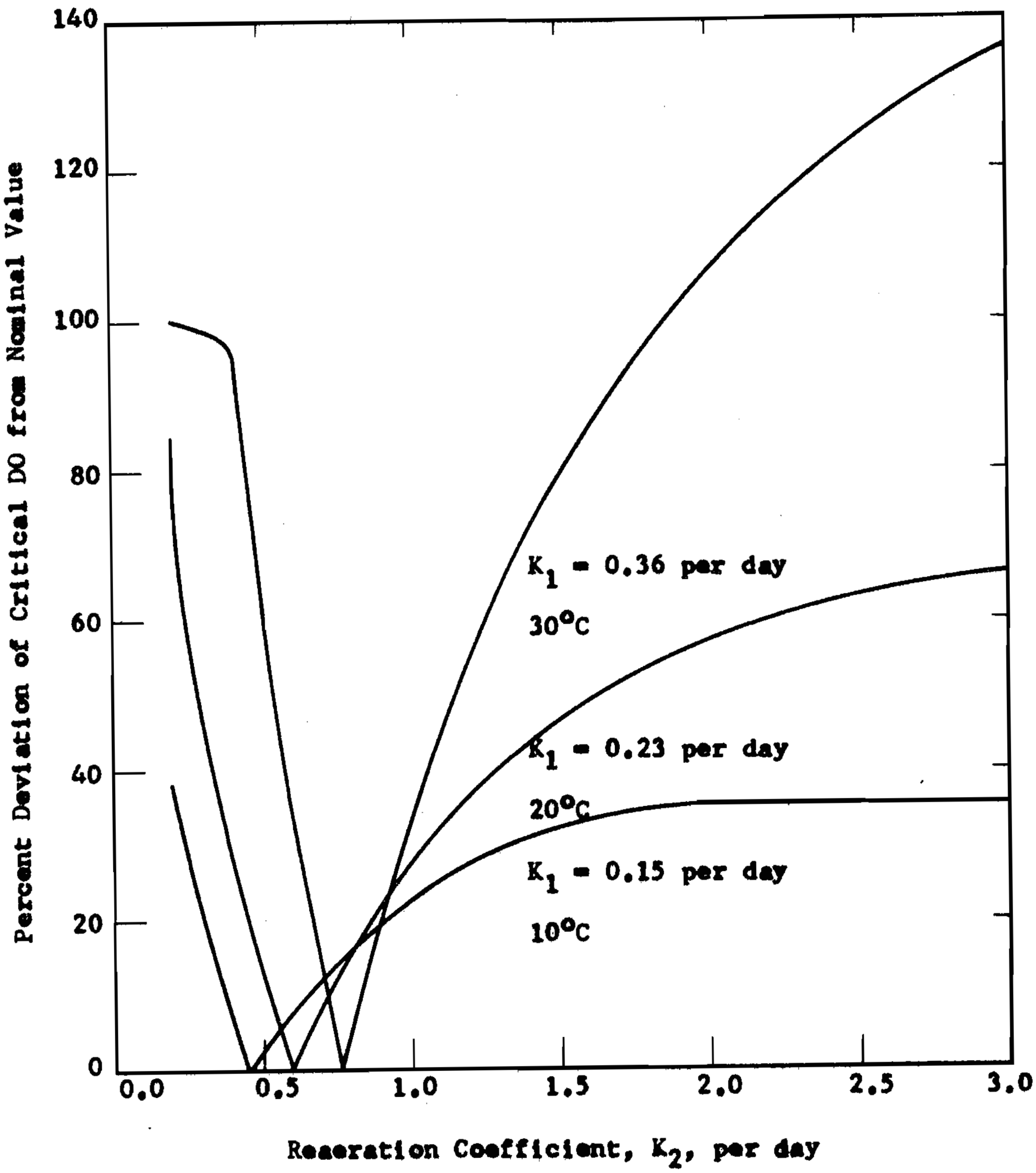

FIGURE 2. EFFECT OF VARIATIONS IN K_2 VALUES ON PERCENT ERROR IN PREDICTING THE CRITICAL DO AT DIFFERENT TEMPERATURES

than the assumed average value, the rate of reaeration will be considerable compared to the deoxygenation rate with the result that the critical deficit in the system will be the initial deficit itself. This upper bound for the percent error in DO prediction varies with the temperature.

Table 5 shows the sensitivity of the system for the variations in K_1 at 10°C, 20°C, and 30°C. These are plotted in Figure 3. Figures 4 to 6 show the relative effects of the deviations in K_1 and K_2 on the sensitivity of the system at temperatures 10°C, 20°C, and 30°C respectively. The curve marked as "lower scale" indicates the sensitivity of the system when K_2 varies and all other parameters of the system remain constant. Likewise the curve marked as "upper scale" refers to the sensitivity of the system when K_1 varies while other parameters remain constant. It is seen from these curves that except at 10°C, the variations in the velocity coefficients are equally significant in predicting the critical DO. At 10°C, the error in prediction when K_1 takes values greater than its nominal value is much more than the error due to the variations in K_2.

Generally, it is seen that the errors in prediction due to the variations in the reaction coefficients are considerable, with the result that there is no justification for using average values for these parameters in predicting dissolved oxygen concentrations. Hence it is postulated that by treating the velocity coefficients as variable coefficients instead of as constant coefficients, the values predicted by mathematical modeling can be made to approach the true value for DO

TABLE 5

EFFECT OF VARIATION IN K_1 ON PERCENT ERROR IN PREDICTING CRITICAL DO AT DIFFERENT TEMPERATURES

Temperature, °C	Saturation DO, mg/l	K_1, per Day	K_2, per Day	Critical Deficit mg/l	Percent Saturation	Percent Deviation from Nominal Value	Remarks
10	11.30	0.10	0.47	3.06	72.92	13.80	
		0.15		4.06	64.08	0.00	Nominal Value at 10°C
		0.20		4.89	56.73	11.47	
		0.30		6.24	44.78	30.12	
		0.40		7.30	35.40	44.76	
		0.50		8.18	27.61	56.91	
		0.60		8.90	21.24	66.85	
20	9.20	0.10	0.60	2.56	72.17	43.71	
		0.20		4.17	54.67	8.86	
		0.23		4.58	50.22	0.00	Nominal Value at 20°C
		0.30		5.41	41.20	17.69	
		0.40		6.40	30.43	39.41	
		0.50		7.22	21.52	57.15	
		0.60		7.93	13.80	72.52	
30	7.60	0.10	0.76	2.15	71.71	129.91	
		0.20		3.55	53.29	70.86	
		0.30		4.66	38.68	24.01	
		0.36		5.23	31.19	0.00	Nominal Value at 30°C
		0.40		5.58	26.58	14.78	
		0.50		6.35	16.48	47.16	
		0.60		7.07	6.97	77.65	

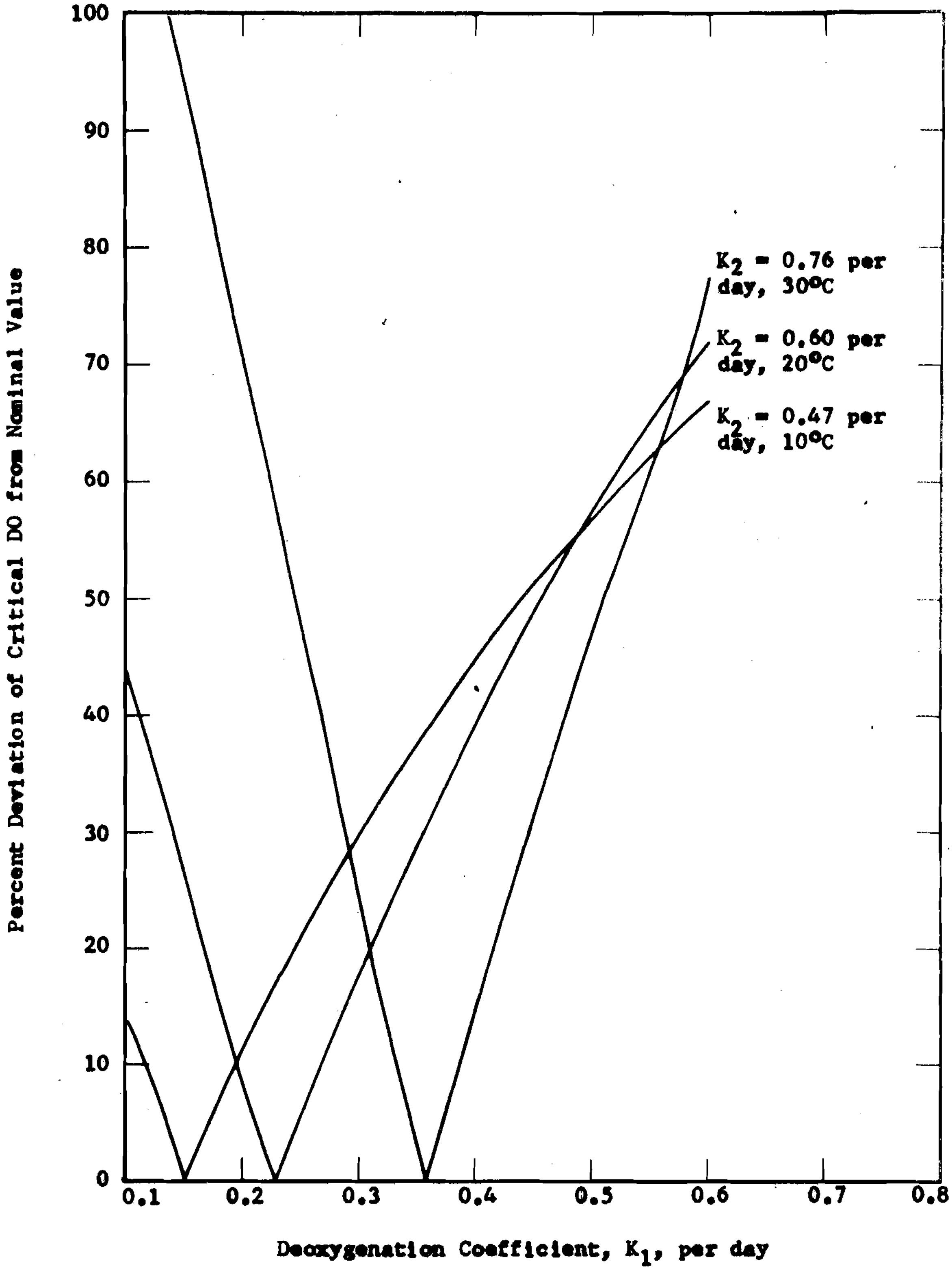

FIGURE 3. EFFECT OF VARIATIONS IN K_1 VALUES ON PERCENT ERROR IN PREDICTING CRITICAL DO AT DIFFERENT TEMPERATURES

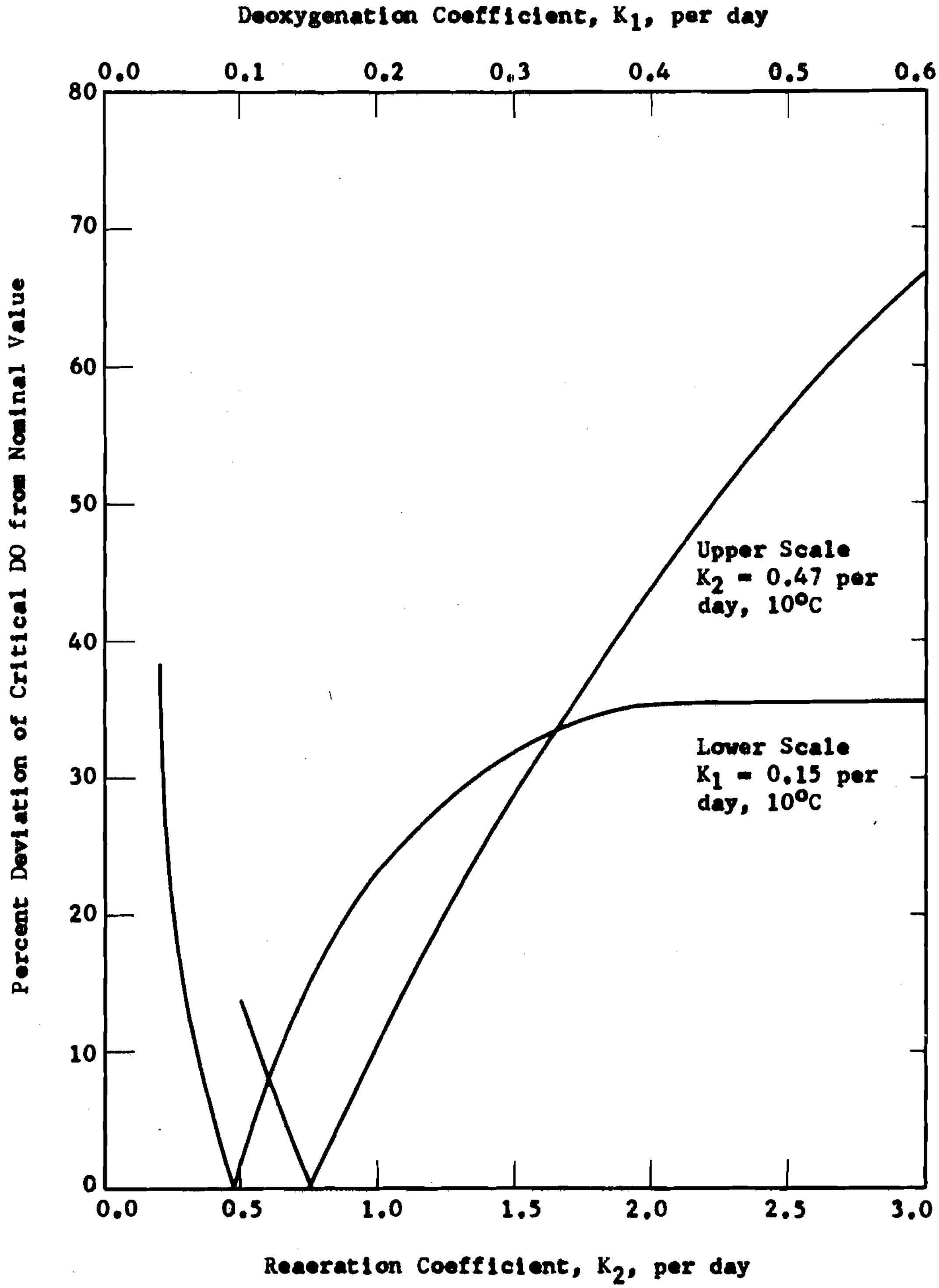

FIGURE 4. COMPARISON OF THE EFFECTS OF VARIATIONS IN K_1 AND K_2 ON PERCENT ERROR IN PREDICTING THE CRITICAL DO AT 10°C

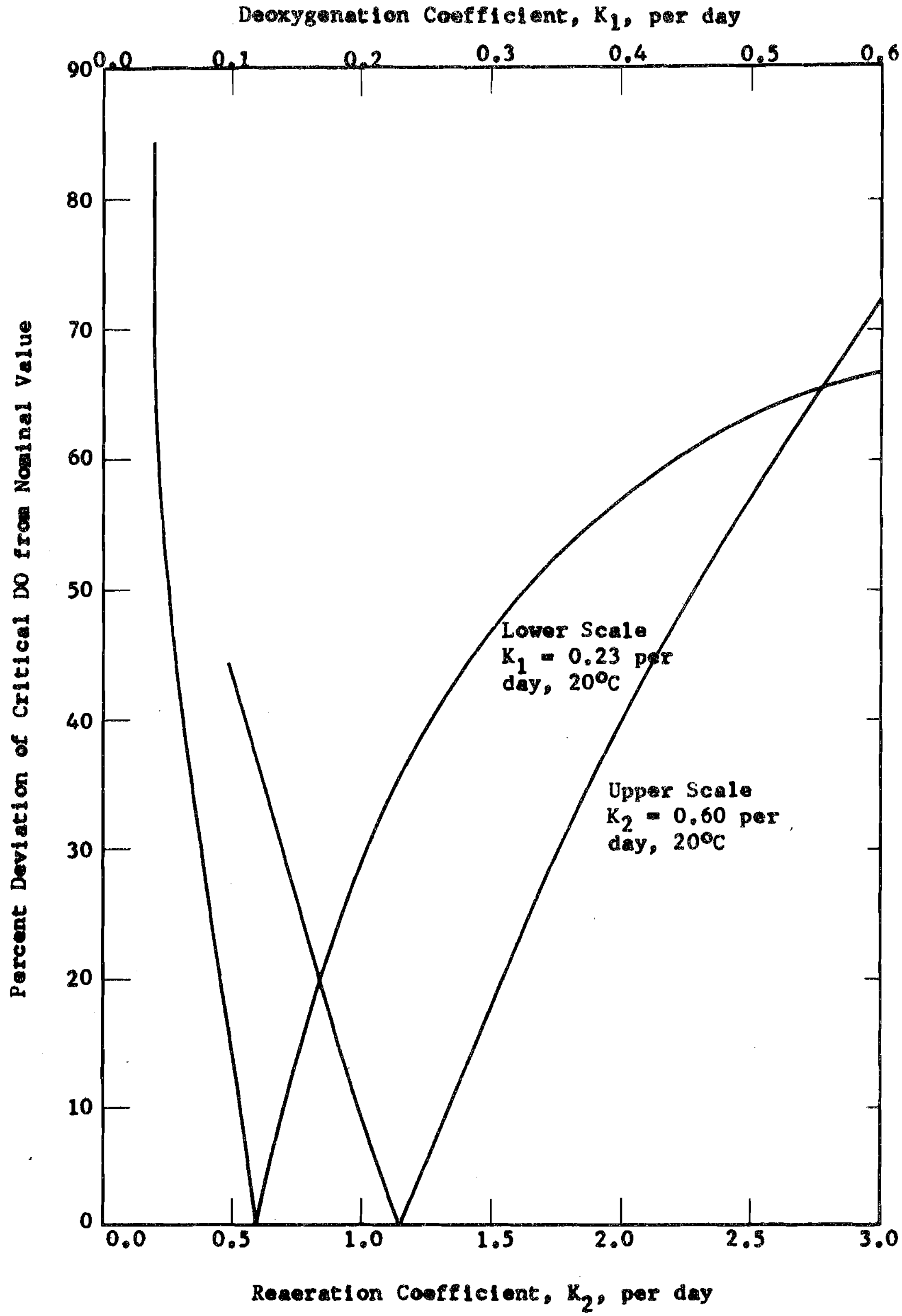

FIGURE 5. COMPARISON OF THE EFFECTS OF VARIATIONS IN K_1 AND K_2 ON PERCENT ERROR IN PREDICTING THE CRITICAL DO AT 20°C

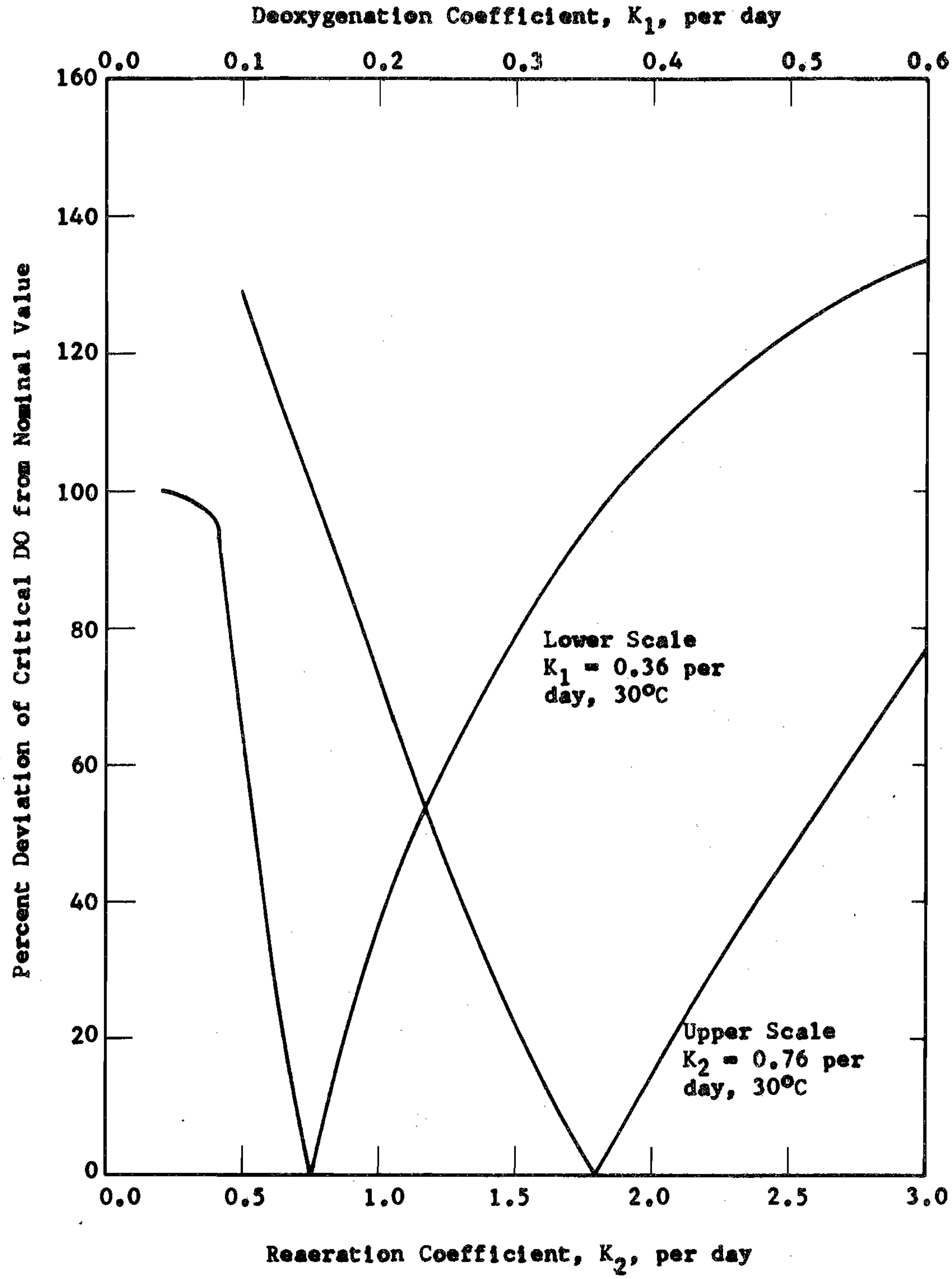

FIGURE 6. COMPARISON OF THE EFFECTS OF VARIATIONS IN K_1 AND K_2 ON PERCENT ERROR IN PREDICTING THE CRITICAL DO AT 30°C

better than it is possible now. The problem of evaluating the assimilative capacities of streams is approached by simulation technique using the Monte Carlo method which makes possible the treatment of the parameters K_1 and K_2 as variable coefficients. The concepts and procedure involved in these will be developed in subsequent chapters.

IV. PROBABILISTIC VARIATIONS IN VELOCITY COEFFICIENTS

It has been found that in many of the practical problems which defy theoretical solutions, an approximate but workable solution could be found using Monte Carlo techniques. In these methods, it is necessary to specify a priori the probability distributions for the random variations of separate events which are then merged into a composite picture. Few attempts have been made to define the chance variations, in terms of probability measure, for the initial BOD and DO at the source of pollution. Montgomery (1964) assumed the daily variations of waste loads, in terms of BOD, to be normally distributed and the assumption was based on considerations given to the histories from Gary, Indiana, Galesburg, Illinois, Dallas, Texas, and Racine, Wisconsin. Whereas, Thayer (1966) in an attempt to formulate a stochastic model for pollution and dissolved oxygen in streams postulated, without verifying the assumption, that the initial BOD and DO in a stream vary independently according to the binomial distribution law. Though it is conceptually feasible, under certain simplified assumptions, to consider all the separate events involved in the mathematical model for stream assimilative capacities as random events, attention will primarily be focused on the variations in velocity coefficients K_1 and K_2 and the proposed probabilistic model will be tested for known initial conditions.

VARIATIONS IN THE DEOXYGENATION COEFFICIENT

As has been mentioned earlier, it is well documented that the deoxygenation coefficients of domestic and industrial wastes at any given location vary considerably and attempts have been made to formulate methods to circumvent the difficulties and inaccuracies arising out of these variabilities (Churchill and Buckingham, 1956; LeBosquet and Tsivoglou, 1950). In order to place the art of predicting stream assimilative capacities on a more rational and time tested basis than those proposed by Churchill and Buckingham (1956), and LeBosquet and Tsivoglou (1950), it is necessary to extend the concepts of Streeter and Phelps (1925) taking variabilities in reaction coefficients into account.

In sanitary engineering practice, it has been tacitly assumed that the characteristics of any waste, particularly those of domestic origin, vary from time to time. In most of the controlled studies, it is customary to use synthetic sewage rather than the waste generated in an actual situation. One of the parameters which define the characteristics of a waste is the rate at which it is biologically oxidized. Since the characteristics of wastes vary considerably from time to time, the rate parameter for the wastes, treated or untreated, also vary considerably. The general course of deoxygenation and the K_1 values vary since biological processes deal with the response of living organisms to their environment and do not show the same degree of uniformity or consistency even under standard conditions, as purely chemical processes do. Even though, this phenomenon can be attributed to the viability and type distribution of microbial population present and the changes in characteristics of the substrate for the microbes, it is not possible to predict the inter-relationship of changes in waste characteristics and microbial response quantitatively in a deterministic sense. Due to the inherent ambiguities involved in the deoxygenation process, where most of the contributing factors are subject to random fluctuations, it is very well suited for being treated under the theories of probability.

K_1 Values of the Ohio River Samples

Whenever one is inferring general laws on the basis of particular observations associated with them, the conclusions are uncertain inasmuch as the particular observations are only a more or less representative sample from the totality of all possible observations. One way of reducing uncertainty in a solution is to collect and base it upon more observations. Data from a 1957 survey of the Ohio River, collected and published by the United States Public Health Service (1960) in connection with the evaluation of the effects of navigational improvements on assimilative capacity of the Cincinnati Pool, is one of the most extensive and comprehensive set of all river pollution survey data published. Extensive information on long-term (20°C) BOD test results for sewage treatment plant effluent and river samples downstream of effluent outfalls, DO test results, photosynthetic oxygen production ("Light and Dark Bottle") test results, etc., are presented. For purposes of studying the probabilistic variations of the rate coefficient K_1 in an actual situation, the long-term BOD data on river samples collected in the reach between miles 474.6 and 481.45 is used in this study. All the pollutional loads emanating from Cincinnati enter the river above this reach. The long-term BOD data on river samples are abstracted from the published data and presented in Appendix D.

Having the BOD data available, the next step was to compute the monomolecular curves which best fit the observed data. It was noted earlier that, because of its traditional usage and general acceptance, it will be assumed that the first stage of deoxygenation proceeds according to a monomolecular or first order reaction. Some method had to be selected of the numerous ones reported in the literature for the evaluation of the doxygenation coefficient and the corresponding ultimate BOD.

One of the earlier methods for the calculation of the velocity coefficient K_1 was that proposed by Reed and Theriault (1931) which uses the least squares principle for minimizing the residual errors between the observed data and the fitted monomolecular curve. In the application of the procedure, a method of successive approximations involving trial and error is used in which it is necessary to assume a trial value of K_1, and this is then used together with the observed data in determining a correction factor to the assumed K_1 value. The operation is repeated with the corrected K_1 value until the correction factor is small, or in other words, until the assumed and calculated values of K_1 approach one another. Obviously if the initial estimate is good, the number of iterations will be minimal, but even if this estimate is poor the procedure will eventually yield a good estimate of K_1. The time necessary for hand computation is excessive and this no doubt had been a factor in the development of less rigorous computational and graphical procedures (Fair, 1936; Moore, Thomas, and Snow, 1950; Thomas, 1937). Since it is generally considered (Gannon, 1963; Zanoni, 1967) that the Reed-Theriault method affords the most reliable estimation for K_1 and ultimate first stage BOD, this method is used in this study in evaluating these parameters for the observed BOD data. The flow diagram for the computer program is shown in Figure 7, and the tolerance limit between the assumed and corrected values for K_1 is set at 0.005 per day. The terminology used in the flow diagram and in the computer program is the same as that adopted by Reed and Theriault (1931).

In determining the parameters for the first stage carbonaceous demand, it is necessary to make corrections for the initial lag in oxygen uptake and oxygen demand due to nitrification, if these effects are significant in the BOD progression data. Figure 8 shows the long-term (20°C) BOD data for effluent samples from Mill Creek and Little Miami sewage treatment plant. It is readily seen that there is no lag in initial oxygen uptake in either of these samples. Also, the trend in BOD progression deviates on or around the tenth day, heralding the onset of nitrification in the bottle tests. Figure 9 shows the long-term (20°C) BOD progression results for the river samples obtained in the Ohio River-Cincinnati Pool area. Here again, no noticeable lag is discerned and the nitrification process is found to set in on or around the tenth day. The BOD progression curves presented here are typical of the results obtained in the Ohio River survey. Since the flow-through time involved in the survey is never more than four days, oxygen demand due to nitrification is not considered in this study. Also, the BOD progression data observed up to the tenth day were used in the Reed-Theriault method for computing K_1 and ultimate first stage BOD values. In most cases there were six observations within ten days in each series

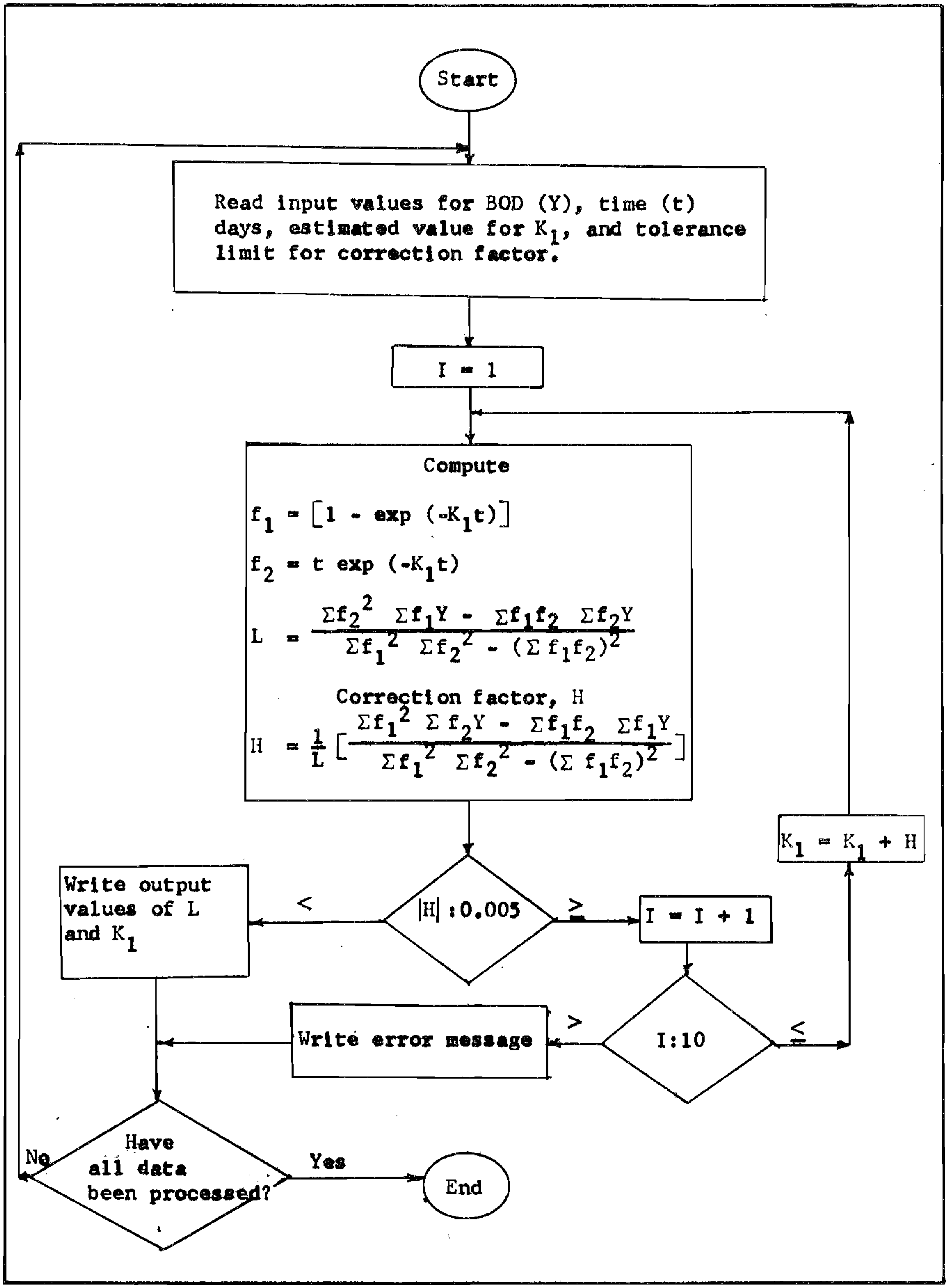

FIGURE 7. FLOW DIAGRAM FOR COMPUTING K_1 AND ULTIMATE FIRST STAGE BOD VALUES USING REED-THERIAULT METHOD

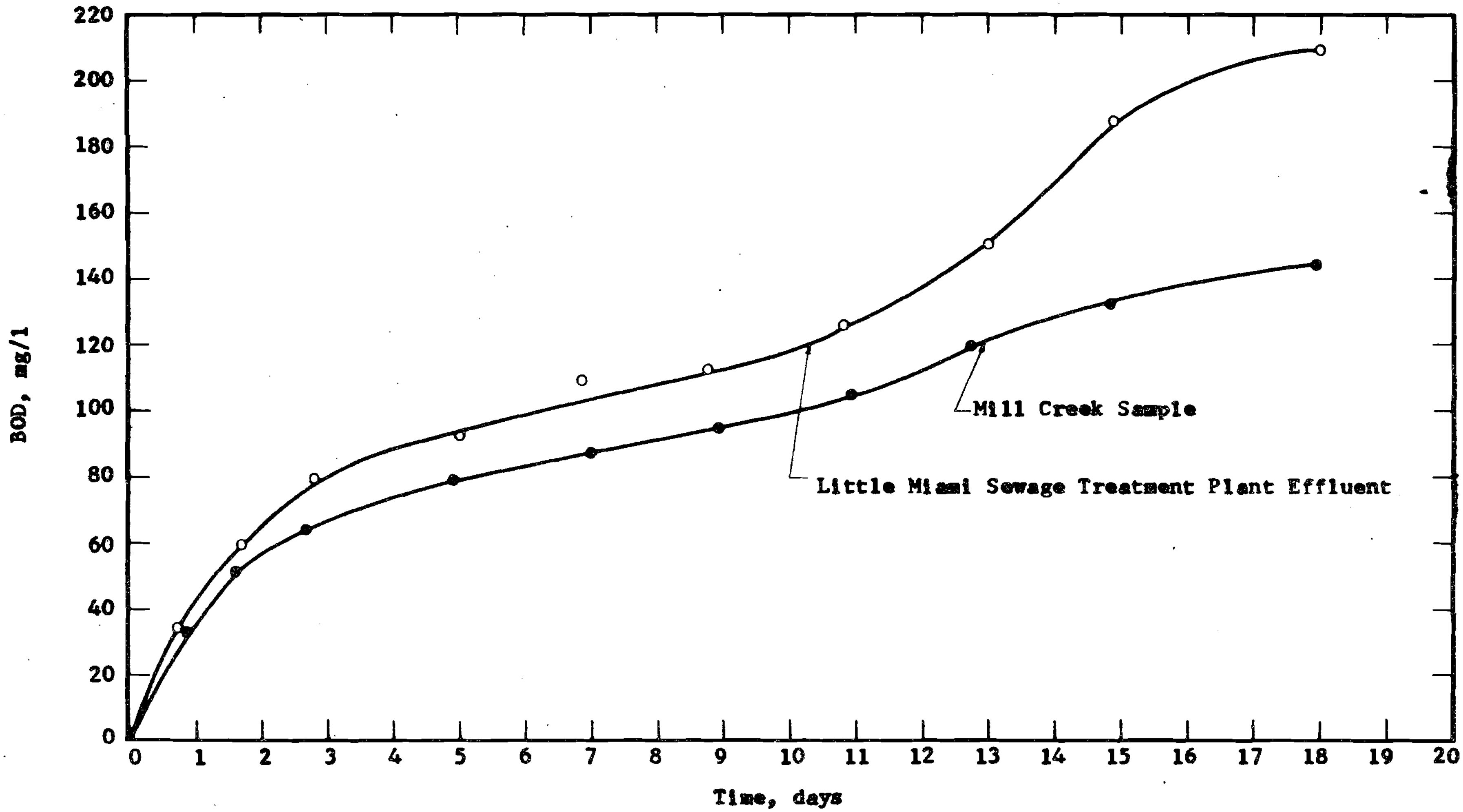

FIGURE 8. BOD PROGRESSION OF TREATMENT PLANT EFFLUENTS

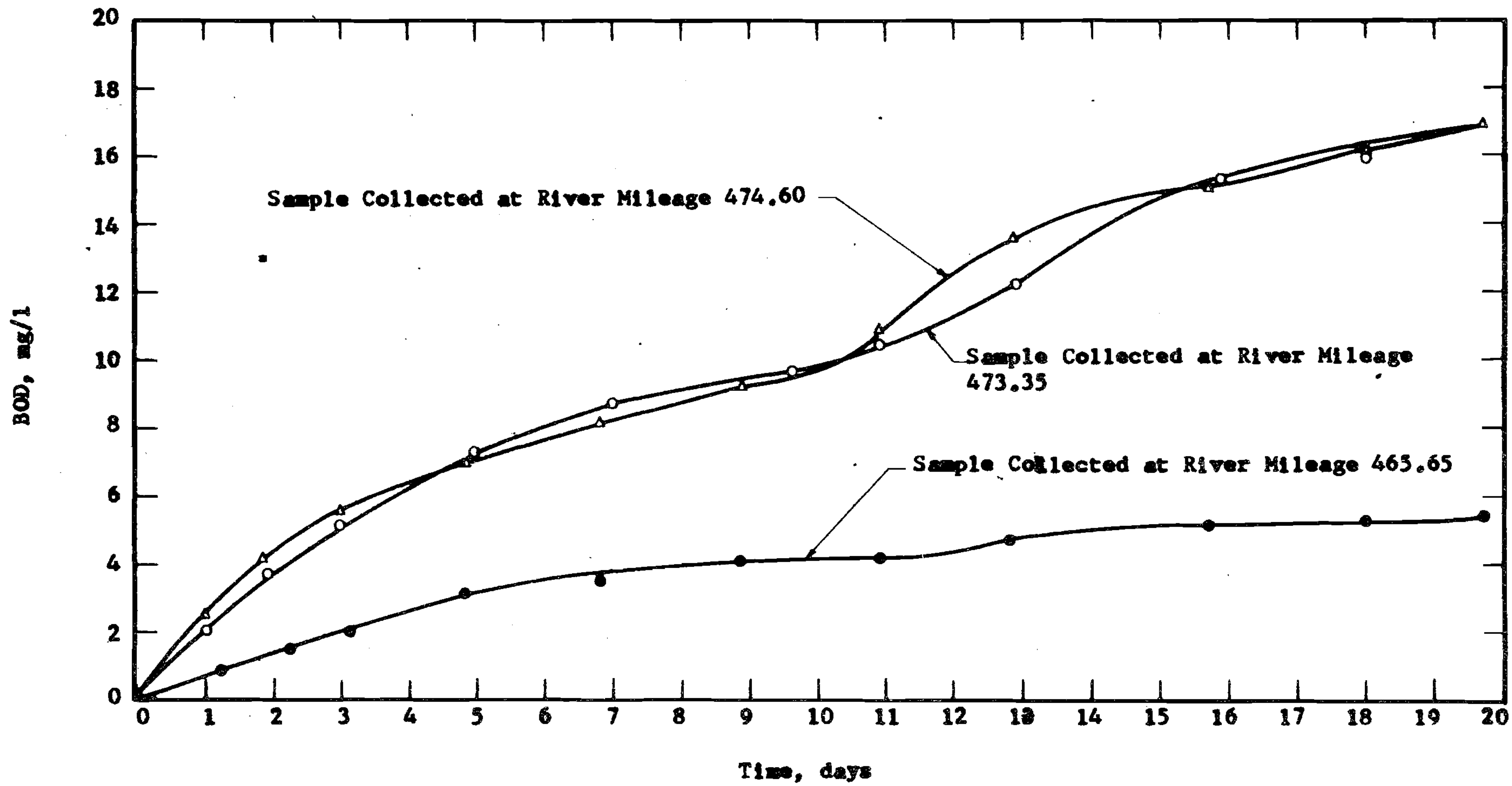

FIGURE 9. BOD PROGRESSIONS OF RIVER SAMPLES

of BOD progression tests and five observations in a very few cases. The values for K_1 (20°C) and ultimate first stage BOD computed for each series of observations on the Ohio River samples are shown in Appendix D.

Randomness Test

Many nonparametric statistical procedures are based on the assumption of a random sample of univariate observations. Also randomness of a set of observations is sometimes itself the property investigated. Although the concept of independent observations, a random sample, is well-known, it is worthwhile to define it here. A sample of n observations, x_1, $x_2, \ldots, x_n$ from a population with distribution function F(x) is called a random sample from that population if

$$P\{x_1,x_2,\ldots,x_n\} = P\{x_1\}\,P\{x_2\}\,\ldots\,P\{x_n\} \qquad (13)$$

where $P\{\cdot\}$ is the probability of observing a value less than or equal to the stated value. When a set of observations is analyzed, the first step is to investigate whether the observations may be regarded as a random sample or not. The next step is to formulate a hypothesis for the type of distribution of the population from which the sample is supposed to be drawn randomly and to estimate values of the parameters of the assumed distribution based on the sample.

The most common case attempted is that showing the relationship of an effect to many causes of which a small number of the causes exert greater influence than do all others. In such a case, when neglected variables, inherent errors, and nonhomogeneity of data have relatively small effects, the relationship between the remaining limited number of variables would indicate a narrow spread around a basic function. Stoltenberg and Sobel (1965) in their study to determine the effects of sampling location, organic loading, tidal stage, and sample collection temperature on K_1 values for various samples obtained in the Delaware Estuary, came to the conclusion based on analysis of covariance that these factors did not have significant effect on K_1 values. Hence it is hypothesized that the variations in K_1 values are generally random, not attributable to any definite cause with certainty and this conjecture was subjected to statistical randomness tests using the observed K_1 values for the Ohio River samples. If the hypothesis fails, then one has to separate the trend in the series of observations made and study the random variations about the observed trend.

Determination of Sequence Order. The first step in applying a test of the random sample hypothesis consists of placing the n observations considered in an appropriate sequence order (Walsh, 1962). The method of assigning the integers 1,...,n to the n observations depends on the experimental situation considered. A common method of assignment is on the basis of the times at which the observations are produced. Then the first observation produced is assigned the integer 1, etc. The observations can be ordered on the basis of location. Also a combination of location and time can be used. In the Ohio River survey, samples from the river reach under consideration were collected for analysis from five locations starting from the head end of the reach and moving downstream to cover all the locations and the whole operation was repeated several times. Since the fate of pollution and the condition of oxygen concentration in the river as the waste discharge flows along the river are of primary importance, the observations for K_1 values are ordered on the basis of the times at which the observations are produced. The sequence of observations thus obtained was then subjected to randomness tests which are discussed in the following sections.

Another approach would be to group the observations according to the place of observation so that any decreasing trend in the K_1 values could be discerned and isolated. It is likely that the K_1 values of river samples decrease in the downstream sections as the more readily degradable organics are oxidized. This approach is not considered in this study since the river sections from which samples were obtained in the Ohio River (USPHS, 1960) are not separated far enough to give any meaningful result.

Runs Up and Down Test. Considering a sequence of n different observations $x_1, x_2, \ldots, x_n$ and the sequence of signs (+ or -) of the (n - 1) differences $(x_{i+1} - x_i)$, a sequence of successive + signs is called a run up and a sequence of successive - signs a run down (Hald, 1960; Walsh, 1962). The length of a run is given by the number of same signs defining the run. The total number of runs is denoted by R, the number of runs of length i by r_i and the number of runs of length k or more by R_k where

$$R_k = \sum_{i=k}^{n-1} r_i \qquad (14)$$

The expected value for the total number of runs E(R), and the variance in the number of runs V(R) are given by

$$E(R) = \frac{1}{3}(2n - 1) \qquad (15)$$

and

$$V(R) = \frac{1}{90}(16n - 29) \qquad (16)$$

Also for $n > 20$, R may be regarded as normally distributed with good approximation. The expected number of runs up and down of length k or more in random arrangements of n different observations for values of k from 1 to 7 are given by Hald (1960) in Table 13.10.

If the hypothesis alternative to the hypothesis of randomness is that a gradual change in the level of distribution has taken place during the drawing of the samples, such a change will produce trends or cycles in the observations, so that one or more long runs may be expected to occur and the total number of runs will be small. The hypothesis of randomness may therefore be tested by means of the total number of runs, a small number of runs being significant and further by means of length of runs, very long runs being significant.

The sequence of observations for K_1 values gives the distribution of runs up and down as shown in Table 6. For a total number of observations of 83 for K_1 values, the expected number of runs and the corresponding variance are given by equations 15 and 16 as 55 and 14.4 respectively. The difference between the observed and expected number of runs is -2. The standard deviation for the total number of runs is 3.8, the variance being 14.4. Hence the deviation of the

TABLE 6

DISTRIBUTION OF RUNS UP AND DOWN FOR THE SEQUENCE OF OBSERVATIONS FOR K_1 VALUES OF THE OHIO RIVER SAMPLES

Length of Run i	Number of Runs Observed Up	Down	Both (r_i)	R_k	Expected Number $E(r_i)$	$E(R_k)$
1	18	15	33	53	34.7	55
2	5	8	13	20	16.3	20.3
3	3	2	5	7	2.9	4.0
4	0	2	2	2	0.9	1.1
≥ 5	0	0	0	0	0.2	0.2
Total	26	27	53	--	55.0	--

observed total number of runs from the expected value is not significant. Also very long runs are not present in the sequence of observations. Hence the hypothesis that the observations for K_1 values form a random sample cannot be rejected.

Turning Points Test. This test consists in counting the number of peaks and troughs in the series (Kendall and Stuart, 1966). A "peak" is a value which is greater than the two neighboring values. If there are two or more equal values which are greater than their predecessor and successor (a rare event in general) they will be regarded as defining one peak. Likewise a "trough" is a value which is lower than its neighbors. Both peaks and troughs are treated as cases of "turning points" and the interval between two turning points is called a phase. The expected number of turning points, P, variance in P, V(P), and the expected number of phases P_i of length i for a sequence of n observations are given respectively as

$$E(P) = \frac{2}{3}(n - 2) \tag{17}$$

$$V(P) = \frac{1}{90}(16n - 29) \tag{18}$$

and

$$E(P_i) = \frac{2(n - i - 2)(i^2 + 3i + 1)}{(i + 3)!} \tag{19}$$

For the river K_1 values, the actual number of turning points in the sequence of observations is 52 as against the expected value of 54.7. The distribution of "phases" for the observed K_1 values are shown in Table 7. The observed values for the total number of turning points and for the "phases" of different lengths are so close to the theoretical values that the hypothesis of random variations in K_1 values cannot be rejected.

TABLE 7

DISTRIBUTION OF "PHASES" FOR THE SEQUENCE OF OBSERVATIONS FOR K_1 VALUES OF THE OHIO RIVER SAMPLES

Phase Length	Number of Phases Observed	Number of Phases Theoretical
1	32	33.3
2	13	14.5
3	5	4.1
4	1	0.9
Total	51	52.8

<u>Hypothesis and Hypothesis Testing</u>

The frequency of occurrence of the observed K_1 (20°C) values for the Ohio River samples in different class intervals are shown in Table 8. The sample mean and standard deviation are respectively 0.173 per day and 0.066 per day with a coefficient of variation of 38.0 percent. The histogram for the observed frequency distributions is shown in Figure 10 along with a theoretical normal curve having a mean and standard deviation of 0.173 per day and 0.066 per day. Figure 11 shows these results plotted on a normal probability paper. It is seen that all the observed points lie very close to a straight line except at the tail ends of the curve fitting where the deviations of the points from the straight line are negligibly small. Since the observed frequency for K_1 values fits very well with the theoretical normal distribution, it is hypothesized that K_1 values for Ohio River samples at Cincinnati Pool, vary randomly according to Gaussian probability with a mean of 0.173 per day and a standard deviation of 0.066 per day. This hypothesis is subjected to further statistical tests as outlined in the following sections.

TABLE 8

FREQUENCY DISTRIBUTION OF OBSERVED K_1 VALUES OF THE OHIO RIVER SAMPLES

Class Interval	Number of Observations	Relative Frequency	Percent Relative Frequency	Cumulative Frequency
0.000 to 0.049	1	0.012	1.2	1.2
0.050 to 0.099	11	0.133	13.3	14.5
0.100 to 0.149	20	0.241	24.1	38.6
0.150 to 0.199	23	0.277	27.7	66.3
0.200 to 0.249	15	0.181	18.1	84.4
0.250 to 0.299	11	0.133	13.3	97.7
0.300 to 0.349	2	0.023	2.3	100.0

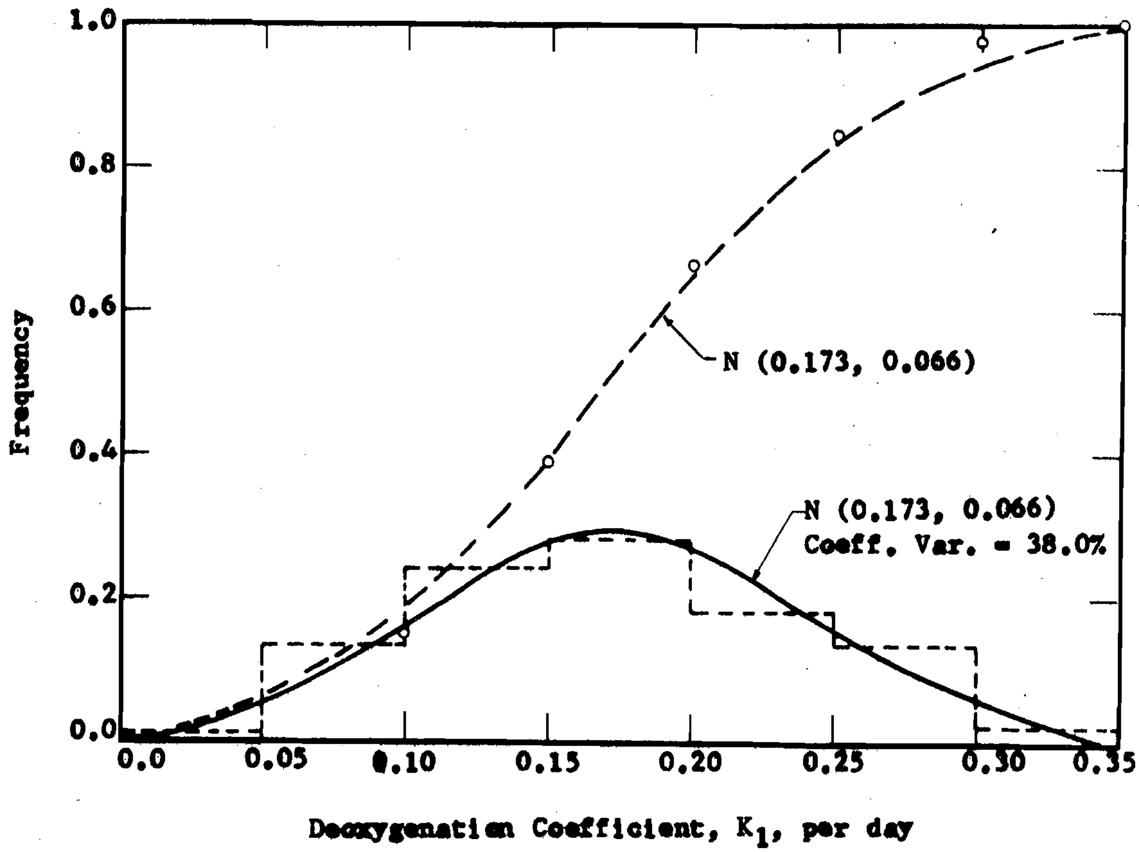

FIGURE 10. FREQUENCY PLOTS FOR K_1 VALUES OF THE OHIO RIVER SAMPLES

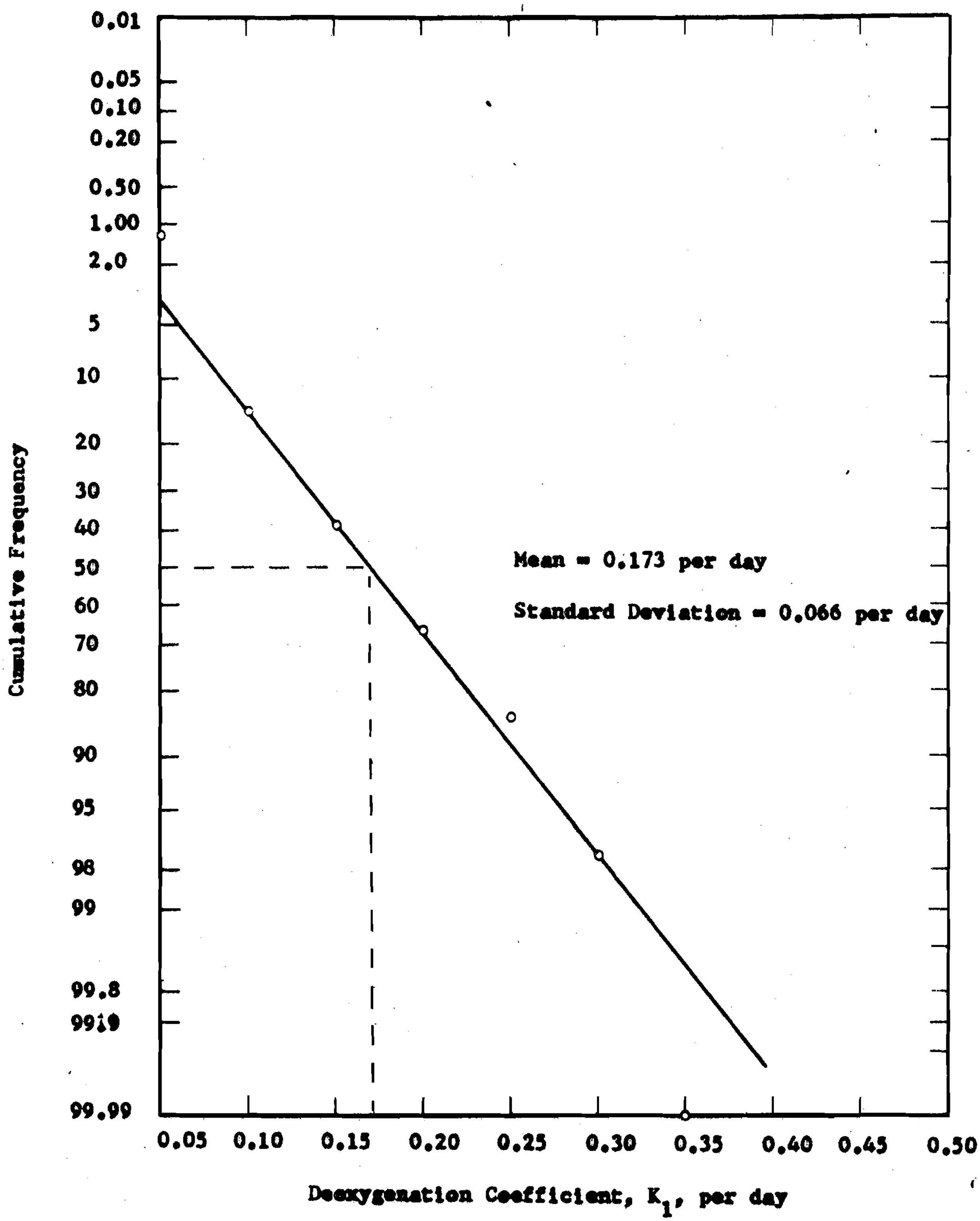

FIGURE 11. CUMULATIVE PROBABILITY DISTRIBUTION OF K_1 VALUES FOR THE OHIO RIVER SAMPLES

Chi-Square Test. This is a simple test generally used for testing whether the data constitute a sample from a population with probability density function (p.d.f.) $f(x)$ at the significance level α (Ostle, 1964). The range of observed values is divided into a number of categories; and the expected number of samples e_i, that will fall in each category i, as predicted by the p.d.f. $f(x)$, is calculated. The observed frequencies n_i in each of the categories are compared with the expected frequencies by computing the chi-square statistic defined by

$$\chi^2 = \sum_{i=1}^{r} \frac{(n_i - e_i)^2}{e_i} \tag{20}$$

Let f be the number of degrees of freedom, given by $f = r - 1 - g$ in which r is the total number of categories, and g is the number of population parameters estimated from the sample and used in theoretical distribution. If the calculated values of χ^2 exceeds the value of $\chi^2_{\alpha,f}$, the hypothesis that the sample belongs to the distribution $f(x)$ is rejected at significance level α.

The result of chi-square test is shown in Table 9. Merging the first and the last of the categories indicated in the table with their respective neighboring categories, the resulting number of categories is 5 and the number of degrees of freedom is 3. Since $\chi^2 = 1.878$ being less than the critical value $\chi^2_{5,3} = 9.35$, the hypothesis cannot be rejected.

TABLE 9

CHI-SQUARE TEST FOR THE GOODNESS OF FIT
CONCERNING THE HYPOTHESIS FOR OBSERVED K_1 VALUES

Class Interval	Observed Frequency (n_i)	Expected Frequency (e_i)	$n_i - e_i$	$(n_i - e_i)^2/e_i$
0.000 to 0.049	1	2.6		
0.050 to 0.099	11	8.5	0.9	0.073
0.100 to 0.149	20	19.0	1.0	0.053
0.150 to 0.199	23	24.6	1.6	0.104
0.200 to 0.249	15	18.3	3.7	0.748
0.250 to 0.299	11	7.8	3.0	0.900
0.300 to 0.349	2	2.2		
				$\chi^2 = 1.878$

Kolmogorov-Smirnov Test. An alternative to the chi-square goodness of fit test is provided by the Kolmogorov-Smirnov test (Massey, 1951; Ostle, 1964). If a population is thought to have some specified cumulative distribution function (c.d.f.) F(x), that is, for any specified value of x, the value of F(x) is the proportion of individuals in the population having measurements less than or equal to x. The cumulative step-function of a random sample of n observations is expected to be fairly close to this specified distribution function. If it is not close enough, it is evident that the hypothetical distribution is not the correct one.

If $S_n(x)$ is the observed cumulative step-function of a sample, i.e., $S_n(x) = k/n$, where k is the number of observations less than or equal to x, then the sampling distribution of d being the maximum of $|F(x) - S_n(x)|$ is taken as the statistic for testing the goodness of fit. The d statistic should be less than the critical value which depends on the number of observations on which the hypothesis concerning the population distribution is based. Massey (1951) has presented proof for the fact that the Kolmogorov-Smirnov test is more powerful than the chi-square test and that this test will detect smaller deviations in cumulative distributions than the chi-square test.

Results of the d test for the K_1 values observed on the Ohio River samples are shown in Table 10. From the table it is seen that the maximum value for the difference between the theoretical and observed cumulative distributions is 0.035. The critical value for the d statistic at 5 percent significance level for a total of 83 observations is 0.149. Since the observed value for the d statistic is less than the critical value, the hypothesis cannot be rejected. The Kolmogorov-Smirnov test, applied to individual observations instead of the grouped data as presented in Table 10, is also satisfied.

TABLE 10

KOLMOGOROV-SMIRNOV TEST FOR THE GOODNESS OF FIT CONCERNING THE HYPOTHESIS FOR OBSERVED K_1 VALUES

Value of the Variable (x)	$S_n(x) = \frac{k}{n}$	Normalized Variable	F(x)	$\lvert F(x) - S_n(x) \rvert$
0.05	0.012	-1.864	0.031	0.019
0.10	0.145	-1.106	0.134	0.011
0.15	0.386	-0.348	0.368	0.023
0.20	0.663	+0.409	0.659	0.004
0.25	0.844	+1.167	0.879	0.035
0.30	0.977	+1.924	0.973	0.004
0.35	1.000	+2.682	0.996	0.004

Since the effect of temperature on the deoxygenation coefficient K_1 has been extensively studied and found to be adequately expressed by the deterministic equation, Eq. 8, it is assumed that the variations in K_1 values at any temperature other than 20°C will also be normally distributed. Also no attempt was made in this study to verify the goodness of fit with other theoretical distributions like lognormal distribution, gamma distribution, t and F distributions, etc. However, the procedure involved in the verification of the goodness of fit with any other theoretical distribution will be the same as that for the normal distribution discussed in this work.

VARIATIONS IN REAERATION COEFFICIENT

Background

The rate of reaeration, under constant conditions of temperature and turbulence, is directly proportional to the oxygen deficit in the water. For many years attempts have been made to derive a method which would permit the prediction of reaeration coefficient, relating some of the easily obtainable hydraulic characteristics of a stream.

O'Connor and Dobbins (1956) presented a theoretical derivation of the reaeration coefficient in which they attempted to define the rate of reaeration in terms of the rate of renewal of the surface film, and assumed that the best estimate was given by the ratio of the vertical velocity fluctuation and mixing length. In streams in which there is pronounced velocity gradient, the following equation for nonisotropic turbulence was developed:

$$K_2 = \frac{480\ D_m^{0.5}\ S^{0.25}}{H^{1.25}} \qquad (21)$$

and for isotropic turbulence, in relatively deep channels where there is no pronounced velocity gradient:

$$K_2 = \frac{127\ D_m^{0.5}\ S^{0.25}}{H^{1.5}} \tag{22}$$

where D_m is the coefficient of molecular diffusion, S is the channel slope and H is the mean depth. Isotropic turbulence was assumed arbitrarily for Chezy coefficients greater than 17 and nonisotropic turbulence for those less than 17. O'Connor and Dobbins (1956) attempted to verify their equations by comparing predicted values of the reaeration coefficients for the Ohio River using the Streeter-Phelps oxygen sag equation (1925). They obtained good agreement in spite of the fact that errors in the estimation of the parameters such as the time of water travel through each reach, the DO concentration, ultimate BOD, deoxygenation coefficient, etc., for use in the Streeter-Phelps equation, are reflected in the computed value of K_2.

Churchill et al. (1962), using multiple regression techniques, determined the coefficients of different equations, formulated by dimensional analysis, relating observed reaeration rates and various hydraulic parameters of river reaches. Corrections for the effects of photosynthesis were made in computing reaearation coefficients from observed measurements of changes in oxygen concentration in selected stream reaches. They concluded that it is possible to predict reaeration rate for a river reach from the average velocity, depth, and temperature of the water using the equation:

$$K_{2(20)} = 5.026\ V^{0.969}\ H^{-1.673} \tag{23}$$

where V is the mean velocity of flow and H is the mean depth. They also suggest that suitable correction should be made for the effects of pollution in predicting stream K_2 values. Inclusion of other stream characteristics such as slope, friction coefficient, and Reynolds number did not markedly increase the accuracy of the predicted value.

A direct correlation between experimentally determined re-aeration coefficients in an artificial channel and the corresponding values for longitudinal mixing coefficient was demonstrated by Krenkel and Orlob (1963). They proposed the empirically derived equation:

$$K_{2(20)} = 1.138 \times 10^{-5} D_L^{1.321} H^{-2.32} \qquad (24)$$

where D_L is the longitudinal mixing coefficient and H is the depth of flow.

Dobbins (1964) proposed a mathematical model for the re-aeration process based on the concept of "an interfacial liquid film" which maintains its existence in the statistical sense, that is the film is always present but the liquid content of the film is being continuously replaced in a random manner by the liquid from the main body. This concept led to the formulation of the equation

$$K_L = (D_m r)^{\frac{1}{2}} \coth \left(\frac{rL_t}{D_m}\right) \qquad (25)$$

in which K_L is the liquid film coefficient, r is the average frequency with which the stagnant film is replaced and L_t is the thickness of film. He further developed several equations using postulated constants to arrive at the relationship between K_L, r, L, and K_2 applicable to natural streams. His attempts to establish numerical values for the hypothesized constants and proportionality factors were somewhat less than successful. Thackston and Krenkel (1965) have raised objection to the correctness of the theory proposed by Dobbins (1964), questioning the validity of the propounded relationships between the constants used in the theory, some of which are based solely on judgment.

Several other attempts to formulate empirical relationships for the prediction of K_2 values in rivers have been reported in the literature. Owens _et al._ (1964) proposed an equation based on the data obtained in several reaches of lowland and Lake District streams in England, using multiple regression analysis as:

$$K_{2(20)} = 10.09\ V^{0.73}\ H^{-1.75} \qquad (26)$$

Also Langbein and Durum (1967) gave the following relationship:

$$K_2 = 3.3\ V\ H^{-1.33} \qquad (27)$$

In all these prediction equations developed either on theoretical basis or using regression analysis, considerable scattering of the observed values for K_2 are noticed about the prediction line. As has been mentioned earlier, in one of the reaches of the Holston River in the Tennessee Valley, the observed K_2 value ranged from 0.10 to 1.18 per day at 20°C for constant discharge in the river whereas the predicted value for the reach was 0.59 per day. Also in another reach of Watauga River which is a shallow and rapid stream compared to Holston River, observed values for K_2 varied from 2.26 to 8.86 per day whereas the predicted value using regression Eq. 23 developed for these rivers was 3.22

per day. Most of the mathematical models developed so far for predicting K_2 values in rivers use at best the average values for river velocity and depth. Since these change from section to section and also within the section itself in natural streams, the estimated average values do not yield satisfactory results. Also errors in sampling and analyses, and in the estimation of mean travel time combined with the uncertainties of wind effects, all complicate the prospects of predicting K_2 reasonably accurately.

Eckenfelder and O'Connor (1961) indicated that the variations in K_2 depend upon the hydraulic properties of the particular channel: roughness, width and curvature, and that the variations are usually defined by a normal distribution. They further suggested that any value of the reaeration coefficient within limited statistical ranges could be used in oxygen balance calculations. O'Connor (1958) presented data, taken from actual stream surveys, showing variations of depths from station to station in the Wabash, Clarion and Codorus Rivers. These were found to be normally distributed and he discussed the importance of considering these variations in determining the required number of cross sectional areas to insure for a given probability that the measured mean depth for use in prediction equations for K_2 will fall within a given percent of the true mean.

Not only the variations in depth affect the evaluation of K_2 values, but also several other factors discussed earlier come into play. It becomes necessary to ascertain the combined effects of all these factors, since it is not possible to assess the effects individually in most cases. The data collected by the Tennessee Valley Authority (1962) on natural streams with constant discharges for each set of observations, being the most extensive, will be used in this study to determine the extent of variations and the parameters characterizing these variations.

Modified Regression Equation

The data, obtained in Tennessee Valley rivers (1962), comprised of 509 individual observations under 30 different sets of experimental conditions. In deriving the regression equations, Churchill et al. (1962) used geometric means of observations for K_2 in each set of experiments. In the knowledge of this author, most of the statistical theories concern themselves with the arithmetic means and not with geometric means. In the application of statistical and probability theories to real phenomena two results play a conspicuous role (Ostle, 1964; Parzen, 1966). These results are known as the Law of Large Numbers and Central Limit Theorem, and deal with the arithmetic means of observations. Hence a regression analysis of the data obtained in Tennessee Valley rivers using arithmetic means of group observations was carried out in this study. An abstract of published data, arithmetic means of observations, and the residual errors in the regression analysis are shown in Appendix E.

A general linear regression equation may be represented by

$$Z_0 = A_0 + A_1Z_1 + A_2Z_2 + \dots A_pZ_p \qquad (28)$$

where Z_0 is the dependent variable estimated from the independent variables $Z_1, Z_2, \dots, Z_p$. From the historical data, Z_i, with $i = 1, \dots, p$ are known and Z_0', the actual value of dependent variable Z_0 is also known. For the least squares linear regression, the coefficients A_0, $A_1, \ldots, A_p$ are so evaluated that the sum of the squared errors

$$S_E^2 = \Sigma (Z_0' - Z)^2 \tag{29}$$

is minimized. Details of solution are available from standard text books on statistics (Ezekiel and Fox, 1959; Ostle, 1964). $Z_0' - Z_0$ is referred to as the residual error, or as the random component, and so the least square regression minimizes the variance of the random component.

The coefficients A_i, with $i = 1, \ldots, p$ are known as the net multiple regression coefficients of Z_0 on Z_i. The coefficient of multiple correlation $R_{0.1,2,\ldots,p}$ is a measure of the proportion of the variance of Z_0 that is explained by the multiple linear regression equation. It is given by

$$R^2_{0.1,2,\ldots,p} = \frac{\sum_{i=1}^{p} A_i \{ \sum_{j=1}^{n} \Delta Z_{0j} \Delta Z_{ij} \}}{\sum_{j=1}^{n} \Delta Z^2_{0j}} \tag{30}$$

In the IBM 7094 system available at the University of Illinois, there is a standard program of the Statistical Services Research Unit for calculating various statistical estimates of multiple linear regression including the regression coefficients, the coefficient of multiple correlation, etc. It was used in this study for multiple regression analysis of the data for river reaeration coefficients.

The regression analysis yielded an equation

$$K_{2(20)} = 5.827\ V^{0.924}\ H^{-1.705} \qquad (31)$$

with a correlation coefficient of 0.917 which is better than the reported correlation coefficient of 0.822 (Churchill et al., 1962) using geometric means for the group data. The form of the regression equation was linearized by taking logarithm of both the sides of the equation before applying the general linear equation, Eq. 28. Results of the regression analysis are shown in Figure 12. There is considerable deviation of the observed values for K_2 from the predicted values.

An analysis of the distributions of percent error indicated that they are normally distributed with mean zero and a standard deviation of 36.8 percent as verified by Kolmogorov-Smirnov test for goodness of fit. Percent error was obtained by dividing the residual error by the predicted value and multiplying the resulting fraction by 100. Figure 13 shows the cumulative probability distributions of percent residual errors in predicting K_2 values, plotted on a normal probability paper. The nature of the distribution of errors in the prediction of K_2 values is not affected by temperature changes, since the basic assumption in any regression analysis is that the errors in prediction are normally distributed.

Since the error distributions are considered random, it is hypothesized that K_2 values for Tennessee Valley rivers could be predicted by superimposing a random component governed by a normal probability distribution with a mean zero and a standard deviation of 36.8

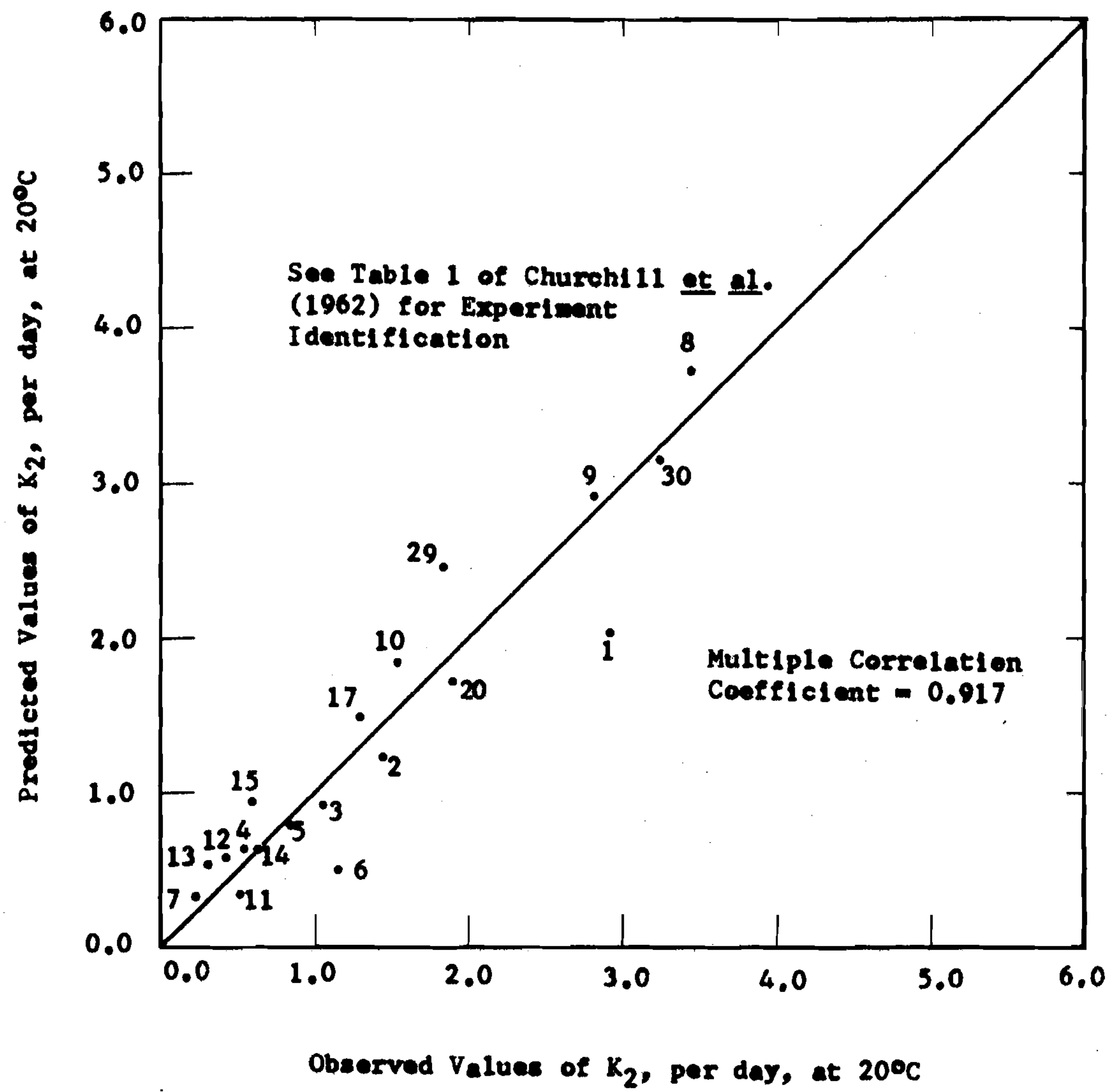

FIGURE 12. OBSERVED VERSUS PREDICTED VALUES OF K_2

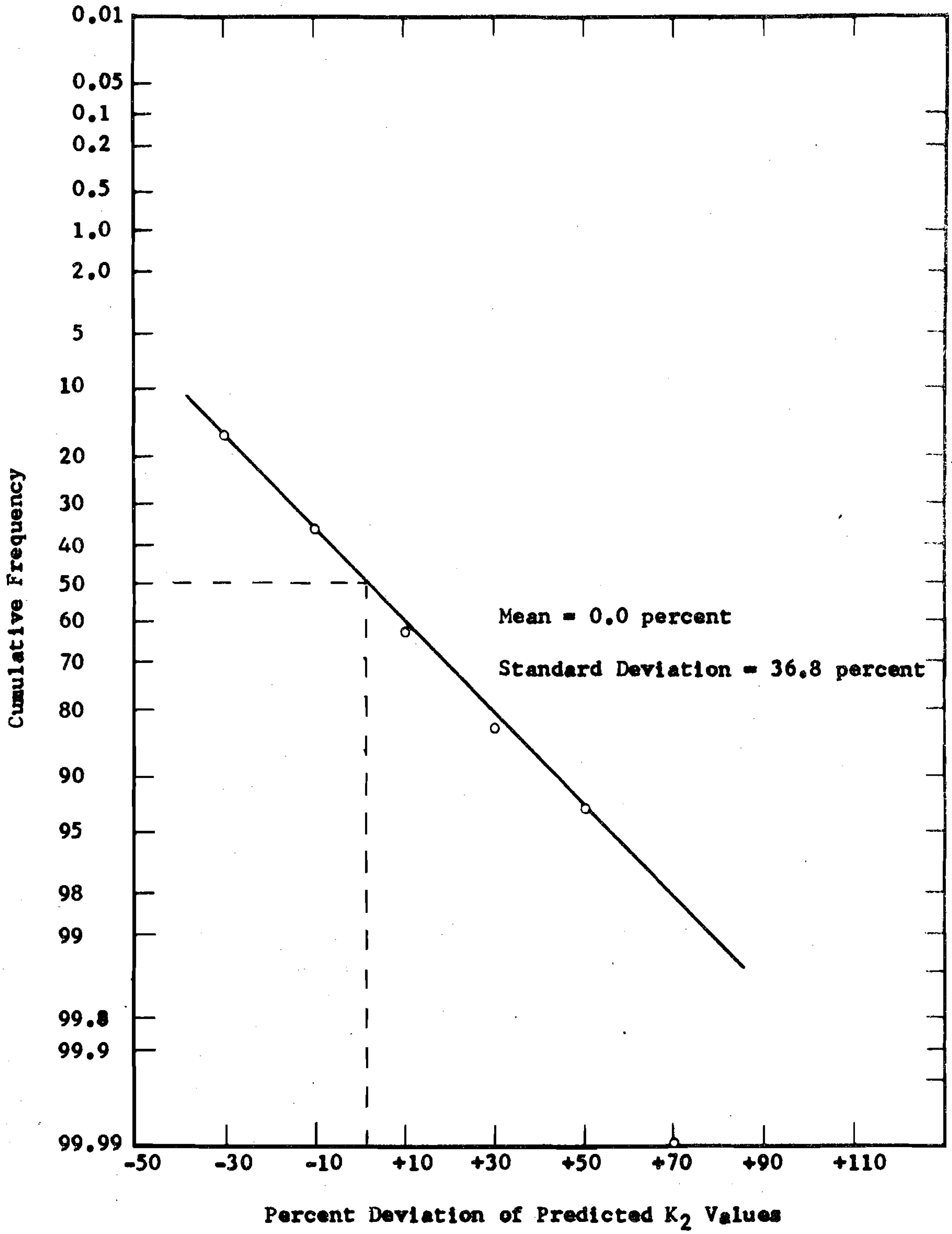

FIGURE 13. PROBABILITY DISTRIBUTION OF PERCENT RESIDUAL ERRORS IN PREDICTING K_2 VALUES

percent, onto a trend component predicted by the regression equation, Eq. 31. Since the standard deviation of the percent error involved in the predictions of K_2 values is considerable, there seems to be little justification in ignoring this random error component altogether in evaluating stream reaeration coefficients.

V. MONTE CARLO METHOD AND SIMULATION MODEL FOR STREAM ASSIMILATIVE CAPACITIES

MONTE CARLO METHOD

General

Monte Carlo methods comprise that branch of experimental mathematics which is concerned with experiments on random numbers (Hammersley and Handscomb, 1964; Hammersley and Morton, 1954; Kahn, 1956; Meyer, 1954). Problems handled by Monte Carlo methods are generally classified into two types; probabilistic or deterministic according to whether or not they are directly concerned with the behavior and outcome of random processes. In the case of probabilistic problems the simplest Monte Carlo approach is to observe random numbers, chosen in such a way that they simulate physical random processes of the original problem, and to infer the desired solution from the behavior of these random numbers. Monte Carlo methods have found extensive use in the fields of nuclear physics and they have been employed in other fields of science like chemistry, engineering, biology, medicine, etc. The significance and the concepts involved in Monte Carlo methods are brought out by considering two examples.

The central problem in designing a nuclear reactor is to determine the critical size of the core. Except when the geometry is simple or the neutrons have small energy ranges, analytical techniques are difficult and recourse is often made to Monte Carlo techniques. A free neutron, put into the core, performs a random walk there, colliding with fissile nuclei until it either escapes from the core or is absorbed by a nucleus. If the former occurs, the neutron continues its random walk in the reflector; if the latter occurs, a fission may result ejecting several free neutrons, called descendants, each of which begins its own random walk. The size of the core is critical when the average number of neutrons in it stays constant. Three probability distributions determine such a random walk (Hammersley and Morton, 1954):

1. The distance a neutron travels between successive collisions is distributed exponentially with a mean called the total mean free path (varying from medium to medium). At a boundary a neutron continues, in the same direction with the same energy, a distance with a mean equal to the total mean free path in the new medium.

2. There are assigned probabilities of the possible types of collision, namely capture, scattering, or fission and their results including energy changes.

3. The direction a neutron takes after a collision is often isotropically distributed, but for collisions with light nuclei is peaked in the direction of the incident neutron's path.

These three distributions depend on the energy of the neutron concerned. Knowing them, one can conduct a neutron from collision to collision by sampling, recording the position, energy, and direction of its motion and its descendants just before each collision. A tally for the number of neutrons in the core is kept due to several parent neutrons in the core as a function of a so-called "census parameter," namely a quantity measuring the current duration of the process (e.g., the number of collisions to date, or the total distance traveled, etc.). This having been done for various core sizes, one can estimate the critical size by interpolating for a constant tally between the increasing and decreasing tallies. The performance of a nuclear reactor is simulated here by choosing random numbers which represent the random motions of the neutron in it. In this way, one can experiment with a reactor without incurring the cost, in money, time, and safety, of its actual physical construction.

As an example for the application of the Monte Carlo techniques in the field of biology, if one wishes to study the growth of an insect population on the basis of certain assumed vital statistics of survival and reproduction, a model could be set up with paper entries for the life histories of individual insects. To each such individual, random numbers are allotted for its age at the births of its offspring and its death; and then treat these and succeeding offsprings likewise. Handling the random numbers to match the vital statistics, one gets what amounts to a random sample from the population which can be analyzed just as real experimental data collected in the field.

Thus it is seen that the Monte Carlo method involves essentially manipulations of random numbers in the simulated model for physical processes and is based on the assumption that in a game of chance, the expected outcome can be estimated in principle by averaging the results of a large number of plays of the game. Also methods employing Monte Carlo techniques have afforded workable solutions in a widely divergent area of endeavors.

There are essentially three steps involved in solving a problem using Monte Carlo techniques. They are:

1. Choosing or analogizing the probability process.

2. Generating sample values of the random variables.

3. Use of variance reducing techniques in the Monte Carlo predictions.

The first of these will be considered in a subsequent section of this chapter in order to maintain the continuity of the discussion on the techniques of Monte Carlo methods. Attention will be focused now on the methods generally used in developing sample values of the random variables.

Random Sampling from Specified Distributions

Sources of Random Numbers. Mechanical sampling devices like disks and roulette wheels have been used for generating random numbers. Since the wheel spinning methods are tedious and cumbersome, they have given way to tables of random numbers and use of computing machines for developing a series of random numbers.

There are several tables of random numbers (The Rand Corporation, 1955; Tippet, 1927; Wold, 1954) which have been obtained by performing permutations on the high order digits of mathematical tables, by compiling from lottery drawings, etc., or from some chance devices. They have been subjected to several statistical tests and are found acceptable for general sampling use. Usually these tables have uniformly distributed decimal digits, though there are some that have Gaussian or normal deviates. In such tables, the first number to be used is generally arbitrarily selected, perhaps by opening a page at random, and with eyes closed, touching a number in the table with a finger. The rest of the numbers are usually chosen in sequence. Random number tables are very convenient for generating a fairly small sequence.

Random Number Generators. Several mechanical or electronic systems have been developed for generating a sequence of random numbers or variables (Tocher, 1963). The electronic machines are based on the principle of converting a source of random noise into a train of pulses which are used to drive a cyclic counter. The drive is interrupted at fixed intervals of time, and the successive status of the counter gives the successive random numbers. Machines that use analog systems in which the random input is self generated are known as analog randomisers. A detailed discussion of the random number generators is given by Tocher (1963).

Development of Pseudo-Random Numbers. Though random number tables are available as punched cards, with increasing use of digital computers, mathematical methods for generating "pseudo-random" numbers within the computing machines have been developed in order to eliminate the need for extensive input of random numbers (Kahn, 1956; Meyer, 1954; Tocher, 1954). Also when it is desired to repeat the calculations for checking purposes, the random numbers used in the initial calculations must be available for the later calculations, and hence pseudo-random numbers are useful for generating a reproducible random number sequence. Though the reproducibility implies the possibility of prediction of the sequence, and hence its nonrandomness, the advantages of reproducibility make it desirable to investigate the possibility of generating approximately random sequences by deterministic processes. Pseudo-random numbers may be defined as reproducible sequence of numbers developed by using a deterministic process that behaves like a random sequence when subjected to certain standard statistical tests. Methods for generating pseudo-random numbers include the mid-square method, the mid-product method, the congruence methods and other miscellaneous methods. Only the multiplicative congruence method which is used in this study is discussed below briefly.

Multiplicative Congruence Method.--One of the characteristics of a pseudo-random sequence is that the sequence is cyclic (Meyer, 1954; Tocher, 1954). Thus attempts have been made to generate cycles of maximum length so that a long nonrepetitive sequence is produced. Lehmer suggested the use of the theory of numbers for developing such long cyclic sequences. The theory of congruences deals with such sequences. In this theory, a number x_1, is said to be congruent with x_2 modulo M if $x_2 - x_1$ is divisible by M.

Consider a first order recursion equation of the form

$$x_{n+1} \equiv kx_n \pmod{M} \tag{32}$$

where k and M are initially arbitrarily chosen. A model given by Eq. 32 is known as a congruential multiplicative model. Then for this model,

$$x_n \equiv k^n x_o \pmod{M} \tag{33}$$

Since, at the end of a cycle of length n,

$$x_n \equiv x_o \equiv k^n x_o \pmod{M} \tag{34}$$

the value of k^n should satisfy the equation

(Kahn, 1956; Tocher, 1954) from a given sequence of uniformly distributed random numbers by using the inverse probability integral transformation. This is based on the basic fact that for any distribution, the cumulative probability distribution function has a rectangular distribution, uniform over (0,1). Let x be the random variable with a cumulative distribution function, denoted by c.d.f., and designated by F(x). The random variable $Y = F(x)$ is given by

$$Y = F(x) = \int_{-\infty}^{x} f(x)dx \tag{38}$$

where f(x) is the probability density function denoted by p.d.f.

Consider the probability distribution of Y. Let the p.d.f. and c.d.f. of Y be g(y) and G(y) respectively. Then,

$$G(y) = \Pr\{Y \leq y\} = \Pr\{X \leq x\} = F(x) = Y$$
$$\text{for } 0 \leq Y \leq 1 \tag{39}$$

and

$$g(y) = \frac{dG(y)}{dy} = 1 \quad \text{for } 0 \leq Y \leq 1 \tag{40}$$

This is the probability density function for the uniform (0,1) distribution. By taking a uniform (0,1) sample of Y, and taking the inverse $x = F^{-1}(y)$ of the probability integral transformation $Y = F(x)$, a sample of X is developed. By repeating this for as many samples as required, a sequence of random numbers having any given distribution can be developed from a sequence of uniform (0,1) random numbers.

The process of inverse transformation can be done by several methods (Tocher, 1954). A relatively simple method for the inverse probability integral transformation is the use of interpolation from a table having corresponding values of x and y, and the use of a suitable degree of polynomial for interpolation. This method, though unsophisticated, is fairly simple and was used in this study to transform the c.d.f. to the random number.

Variance Reducing Techniques

The methods which are most often used to reduce variance in Monte Carlo problems are straightforward sampling, systematic sampling, stratified sampling, use of expected values, correlation, and Russian roulette and splitting (Kahn, 1956; Meyer, 1954). The methods which can be used to reduce variance are often dependent upon the probability model and in some cases on the techniques used to generate values of the random variables. Kahn (1956) has given an excellent discussion of the general nature of the techniques with their applications to evaluation of integrals. The first of these which has found extensive use in engineering problems, and which is used in this study is discussed below.

Straightforward Sampling. This technique is based on the premise that one way or reducing uncertainty in an answer is to base it upon more observations (Hammersley and Handscomb, 1964; Kahn, 1956; Tocher, 1954). Broadly speaking there is a square law relationship between the error in an answer and the requisite number of samples. If a sequence of n random variables $x_1, x_2, \ldots, x_n$ are picked from the p.d.f. f(x) and if a random variable $\hat{z}_n$ defined by the equation and σ^2. It can be deduced from the Central Limit Theorem that

$$\Pr\{\hat{z}_n \leq \bar{z} + \delta\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\delta\sqrt{n}}{\sigma}} \exp\left(-\frac{x^2}{2}\right)dx \tag{44}$$

and that $\sigma^2_{\hat{z}_n}$, the variance of the estimator $\hat{z}_n$ of $\bar{z}$ is given by the equation

$$\sigma^2_{\hat{z}_n} = \frac{\sigma^2}{n} \tag{45}$$

or

$$\sigma_{\hat{z}_n} = \frac{\sigma}{\sqrt{n}} \tag{45a}$$

Thus, it is seen that there is a square law relationship between the error in an answer and the requisite number of observations. To reduce the error tenfold calls for a hundred fold increase in the observations, and so on.

Estimation of Sample Size. In any simulation, it is necessary to know approximately the sample size required before sampling is started in order that the size is neither too large to make it very expensive compared to the information gained, nor too small to be reliable. When the required precision and confidence levels for the reliability of prediction are given, the sample size can be determined by the following approach (Flagle, et al., 1960; Massey, 1951).

Let β and α percent be the error and significance levels for the reliability of predicted values in a simulation model. Within the same confidence interval, the precision could be increased only by increasing the sample size. Since the standard deviation and consequently the tolerance diminishes as the square root of the sample size, to increase the precision by a factor of two, the sample size would have to be quadrupled. If $d_{\alpha(n)}$ is the critical value of the 'd' statistic in the Kolmogorov-Smirnov test for goodness of fit at a signficiance level of α, which is defined as the maximum absolute difference between sample and population cumulative distributions, then the required sample size is given by the equation

$$d_{\alpha(n)} = \frac{c_\alpha}{\sqrt{n}} = \beta \tag{46}$$

where c_α is the constant defining the critical value of the d statistic in the Kolmogorov-Smirnov test for samples of size greater than 35. Knowing the values of α and β, it is then possible to pick out the value of c_α from statistical tables (Massey, 1951; Ostle, 1964) and hence the value of n, being the requisite number of samples to meet the stipulated criteria could be computed.

SIMULATION IN WATER RESOURCES SYSTEMS

Simulation means to duplicate the essence of the system or activity without actually attaining reality itself. Simulation has been used traditionally in engineering. The use of conceptual system models, scale models, analogs, and laboratory experimentation are but some of the general simulation techniques used in engineering. However, this study deals with the simulation of processes involved in the prediction of a stream's waste assimilative capacities using a digital computer. The approximation, complexity, accuracy and cost of simulation are somewhere between those of mathematical analysis and prototype testing. Simulation is especially adaptable to the study of complex systems and in a way, it may be considered as a numerical method for the solution of complicated probabilistic or stochastic processes. It is used to circumvent the difficulties of duplication of environment, of mathematical formulation, of lack of analytic solution techniques, or of experimental impossibilities.

Analog and digital computers are frequently used for predicting the responses of certain water quality parameters in surface water bodies like rivers, lakes, estuaries, etc. With the advent of modern electronic computers, more and more efforts are being directed toward water resources planning on a regional basis, with optimization of water and waste water treatment processes taken as a whole instead of treating these two as separate entities as has been done in the past. Since water resources systems are usually complex with multiple alternatives for design, allocation, and operation, simulation seems to be a very useful tool in water resources system design.

When the mathematical model for a process has been decided upon, various elements of the process can be represented in the computer so that the outputs of one part of the system constitute the inputs into one or more other elements. A complex system may thus be represented by a series of relatively simple elements. In such a system, it is possible to vary the parameters of the system to investigate some specific aspects of the system, and when necessary, to adopt systematic or trial and error procedures for the optimization of the system. Because of these facilities, digital and analog computers have been used extensively for the simulation of dynamic physical systems.

This study deals with the simulation of a river system using a digital computer with specific reference to the simulation of the phenomenon of a stream's assimilative capacity for the organic waste discharged into it without creating nuisance conditions in the stream. This differs from analog models in that the generated data are used for the simulation of the process facilitating the study of the probabilistic aspects of the process as a whole. Generated data, as used here, will refer to generating random values for variables in the process under consideration on the basis of statistical models for the variables, which could not be distinguished from the real or historical data by means of the usual statistical tests of significance. The generated data did not occur in the past, nor will they occur in the future; but based on certain statistical considerations, it could have been the real record. Regarding validity of simulation Flagle _et al._ (1960) state:

> ...It must be remembered that simulation yields only an empirical form of knowledge, fraught with the danger that the stochastic model is not truly representative. In this sense it is admittedly inferior to mathematical techniques which yield functional relationship between the variables. Ultimately we hope that the more slowly developing mathematical capabilities will make simulation as we practice it, unnecessary. Until that time comes, however, simulation techniques stand as powerful aides to the operations analyst who must produce useful results.

MODEL FOR STREAM ASSIMILATIVE CAPACITY

Factors Affecting DO and BOD

Since Streeter and Phelps (1925) propounded their theory defining two mechanisms which affect the DO and BOD relationship namely, the BOD removal due to bacterial oxidation and oxygen addition due to atmospheric reaeration, several other factors affecting DO-BOD relationship have been postulated (Camp, 1963; Dobbins, 1964; Gannon, 1963; Thomas, 1948). Some of the major factors which affect the assimilative capacity of a stream in general can be summarized as follows:

1. Removal of BOD by bacterial oxidation of both carbonaceous and nitrogenous matter.

2. Removal of BOD by sedimentation or adsorption.

3. Addition of BOD along a stretch by the scour of bottom deposits or by the diffusion of partly decomposed organic matter from the benthal layer into the overlying water.

4. Addition of BOD along the stretch by local run off.

5. Biological removal and accumulation of BOD by fixed plants and algae.

6. Addition of oxygen from the atmosphere.

7. Addition of oxygen by the photosynthetic action of plankton and fixed plants.

8. Removal of oxygen by the respiration of plankton and fixed plants.

Thomas (1948) introduced a rate constant K_3, with units of per day, as a means of accounting for the removal or addition of BOD by deposition or resuspension. The net rate was assumed to be proportional to K_3L, a negative value of K_3 indicating deposition and a positive value indicating resuspension.

In considering the stabilization of organic matter in natural streams, distinction should be made between those factors which affect its removal without necessarily utilizing oxygen and those which simultaneously utilize oxygen while removing the organic matter. In the first category are such phenomena as sedimentation, scour, and biological accumulation and the second category includes various forms of chemical and biological oxidation. Generally when a waste is discharged to a stream, it is extremely difficult to isolate the effects due to each of these factors individually. O'Connor (1967) introduced the parameter K_r, with units of per day, to define the total rate of removal of BOD in the river. It is determined from a series of measurements of the BOD at a number of stations downstream from the outfall. Since the rate of removal of organic matter is not necessarily equal to the rate at which oxygen is utilized, the difference between the BOD removal rate and oxygen utilization, if such a difference exists, may be attributed to sedimentation, scour, benthal demand, and biological accumulation. If none of these factors is present or significant, it is usually assumed that the rate at which the organic matter is removed is equal to the rate at which the dissolved oxygen is utilized (O'Connor, 1967), in which case $K_1 = K_r$.

The sources of oxygen in a river stretch are the amount of DO in the incoming flow, that due to natural or artificial reaeration and that due to the photosynthetic addition. The rate of reaeration is proportional to the dissolved oxygen deficit. The photosynthetic source depends upon many factors such as sunlight, temperature, mass of algae and nutrients. If the effect of these factors is included in the term $P_{t'}$, representing the overall rate at which oxygen is released by photosynthesis and if the photosynthetic rate if assumed to vary as the sunlight, reaching a peak at noon and a zero value at sunrise and sunset, then this source could be defined by a periodic function (O'Connor, 1967) as:

$$P_{t'} = \begin{cases} P_m \sin \frac{\pi}{p} t' & \text{when } 0 \le t' \le p \\ 0 & \text{when } 1 - p \le t' \le 1 \end{cases} \tag{47}$$

where p is the period of oscillation of the periodic function, P_m is the amplitude of the periodic function and t' is the time of flow in days of a mass of water since sunrise after the mass of water has entered the head end of a river reach considered. If the period p is twelve hours or half a day, the periodic function shown in Eq. 47 reduces to

$$P_{t'} = \begin{cases} P_m \sin 2\pi t' & \text{when } 0 \le t' \le 1/2 \\ 0 & \text{when } 1/2 \le t' \le 1 \end{cases} \tag{47a}$$

This periodic function can be described by a Fourier series as defined below:

$$P_{t'} = a_o + \sum_{n=1}^{\infty} a_n \cos nt' + \sum_{n=1}^{\infty} b_n \sin nt' \tag{48}$$

The coefficients of the Fourier series can be evaluated using Euler's formula (Kreyzig, 1962) giving

$$a_o = \frac{P_m}{\pi} \tag{49}$$

$$a_n = \frac{P_m}{2\pi}\left(\frac{-\cos(1+n)\pi + 1}{1+n} + \frac{-\cos(1-n)\pi + 1}{1-n}\right) \tag{50}$$

When n is odd

$$a_n = 0 \tag{50a}$$

and when n is even

$$a_n = -\frac{2P_m}{(n-1)(n+1)\pi} \tag{50b}$$

Also

$$b_1 = \frac{P_m}{2} \tag{51}$$

and

$$b_n = 0 \text{ for } n = 2, 3, \ldots \tag{51a}$$

Hence

$$P_{t'} = \frac{P_m}{\pi} + \frac{P_m}{2} \sin 2\pi t' - \frac{2P_m}{\pi}\left(\frac{1}{3}\cos 4\pi t' + \frac{1}{15}\, 8\pi t' + \ldots\right) \tag{52}$$

Thus it is seen that the oxygen addition due to photosynthesis can be represented by the first three terms of Eq. 52 with good approximation. This approach of expressing quantitatively the diurnal variations in photosynthetic activity is decidedly superior to averaging over the entire period including the nighttime flow conditions as was done by Camp (1963) and Dobbins (1964).

Equations for BOD and DO Profiles

The equations for the BOD and DO profiles along a river stretch are based on the following assumptions:

1. The stream flow is steady and uniform.

2. Removal of BOD by both bacterial oxidation, sedimentation or adsorption or both and biological accumulation are first order reactions, the rates of removal being proportional to the amount present.

3. The removal of oxygen by benthal demand, by plant respiration and the addition of BOD from the benthal layer are uniform along the stretch.

4. Diurnal variations of oxygen addition due to photosynthesis can be represented by a sine function.

5. The rate coefficients affecting the oxidation of organic matter and atmospheric reaeration can be taken as random variables and these could be defined by suitable statistical models.

6. The BOD and DO are uniformly distributed over each cross section so that the equations can be written in the one-dimensional form.

Under the foregoing assumptions, the differential equation for the BOD profile is given by

$$\frac{dL}{dt} = -(K_1 + K_3 + K_b)L \tag{53}$$

where K_b is the coefficient for determining the rate of BOD removal due to biological accumulation. The differential equation for the oxygen profile is given by

$$\frac{dD}{dt} = K_1L - K_2D - P_m \left(\frac{1}{\pi} + \frac{1}{2} \sin 2\pi t' - \frac{2}{3\pi} \cos 4\pi t'\right) + R + D_b \quad (54)$$

where R is the rate of oxygen removal by algal respiration and D_b is the benthal demand both with the units of mg/(1)(day). When these two equations, Eqs. 53 and 54, are solved simultaneously with known initial conditions for the BOD and DO at the upper end of a river section, it is possible to predict the state of DO at any point within the reach.

Application to the Problem under Study

Generated data for the values of K_1, K_2 and DO found necessary for the probabilistic analysis of the process under investigation were developed by using Monte Carlo methods and simulation techniques. The statistical models as developed in this study for K_1 and K_2 were used in generating data applicable to the Cincinnati Pool of the Ohio River prior to the construction of Markland Dam. Parameters pertaining to other mechanisms namely photosynthesis, respiration, etc., are evaluated from the published Ohio River Survey data (USPHS, 1960). These parameters were taken at their average values and the expected and modal values for DO as predicted by the probabilistic model for known initial conditions of BOD and DO deficit were compared with the actual DO concentrations observed in the survey.

Using a subroutine available at the Digital Computer Laboratory at the University of Illinois, two sets of uniformly distributed random number sequences were generated and transformed to correspond to normally distributed random variables with given characteristics. The first set of generated values was used to evaluate K_1 and the second K_2. In the case of K_1, the generated normally distributed values with parameters matching the historical values themselves constitute the simulated values for K_1. In the case of K_2, the generated values constitute the random components of K_2 and when these are added to the trend component as given by the regression equation, Eq. 31, the random variable K_2 is obtained. The generated data were tested to see whether the sample could be distinguished from the theoretical model using the Kolmogorov-Smirnov test. The generated values for K_1 and K_2 along with other parameters as determined in the Survey (USPHS, 1960) were used in the simulation model to obtain probabilistic estimates of DO at the desired sections in the river reach considered.

It is appropriate to quote what Flagle _et al_. (1960) had to say on probabilistic approaches for solving problems which do not strictly conform to deterministic rules:

> ...as soon as human participation in the system operation occurs it is necessary to introduce arguments of probability and calculate, not how the system will certainly and invariably perform, but only how it will perform a certain fraction of the time it is stated. Sometimes, as in the case of the atomic bomb already mentioned, statistical methods must be used even for a mechanical system when the laws of physics that apply (like those for radio-activity) are inherently statistical in nature.

This study is limited to the development and subsequent testing of the generated K_1 and K_2 data and the analysis of some of the probabilistic characteristics of the DO response in stream assimilative capacity determinations. Future studies may deal with the simulation of systems of waste treatment facilities, and the combination of water and waste water treatment facilities for determination of the optimal design.

VI. APPLICATION OF MONTE CARLO METHOD TO THE OHIO RIVER SURVEY DATA

CHOICE OF PUBLISHED RIVER SURVEY DATA

The primary objective of this study is to develop a method for estimating the stream's waste assimilative capacity taking into account the variability of the reaction coefficients affecting the DO-BOD relationship; the need for which has long been felt. In order to express quantitatively the variability in these coefficients using probability measure, extensive data are needed and published data for five river surveys (Gannon, 1963; The Resources Agency of California, 1962; TVA, 1962; USPHS, 1960; USPHS, 1963) were examined to study in detail the probabilistic variations in these coefficients and to verify the observed data of dissolved oxygen with the predicted values using Monte Carlo methods. Though the Sacramento River survey (The Resources Agency of California, 1962) was extensive, encompassing information on physical, chemical, and biological aspects, photosynthesis, respiration, etc., the number of observations made for long-term BOD progression below a single major waste discharge was far less than that made in the Ohio River survey (USPHS, 1960), which was otherwise comparable to Sacramento River survey in various other aspects observed. The long-term BOD experiment results published by Gannon (1963) for the Clinton and Tittabawassee Rivers were carried out at different conditions of incubation temperature, mixing, dilution and nitrification inhibition agents; whereas in the Ohio River survey tests for the BOD progression river samples were all carried out according to the procedures stipulated in Standard Methods (APHA, 1955). Also no attempts were made in this survey (Gannon, 1963) to evaluate the effect of photosynthesis due to aquatic vegetation, even though the author concluded that the abundance of aquatic vegetation was responsible for a manifold increase in the river BOD removal rate compared to the BOD removal rate observed in bottle experiments. The report on the Illinois River System (USPHS, 1963) did not include details of the BOD progression data and the number of experiments carried out in this study to evaluate K_1 values was too few to make reliable statistical inference. Hence it was observed that the data published for the Ohio River-Cincinnati Pool survey (USPHS, 1960) which included long-term BOD results on river samples, dark and light bottle tests for determining photosynthesis and respiration of phytoplanktons, benthal deposits, hydraulic characteristics of the river, etc., are best suited for this study.

In any river survey for determining the pollution assimilative capacity, it is necessary to determine the river reaeration coefficient, K_2, independently. In order to estimate the random values of K_2 for the Ohio River-Cincinnati Pool reach, it is assumed that the regression equation for the trend component and the probability distribution function for the random component of K_2 as developed for Tennessee Valley rivers are applicable to the Ohio River reach under consideration.

VALUES OF THE PARAMETERS USED IN THE MODEL

Deoxygenation Coefficient

With the advent and increased use of secondary treatment facilities for waste disposal, the importance of considering oxygen demand due to nitrification has been recognized. O'Connor (1967), and Stratton and McCarty (1967) proposed mathematical models to account for the nitrogenous oxygen demand in oxygen balance studies in rivers. As indicated in Chapter IV, in the case of the Ohio River study, neither the treatment plant effluents nor the river samples obtained below the waste outfalls showed any significant nitrification for about 10 days. Though the mathematical model proposed to be used for the Ohio River does not contain terms to account for nitrification, conceptually it does not present any difficulty to include terms to account for the oxygen consumption due to nitrification for situations where this effect is significant. Thus, the only factor which affects BOD removal with concomitant removal of oxygen is that due to the biological oxidation of organic matter. The details of the analysis of BOD progression data obtained on Ohio River-Cincinnati Pool samples were presented in Chapter IV. The variations of K_1 values for these river samples were found to be random and adequately defined by a statistical model; namely, that the K_1 values for river samples were normally distributed with a mean of 0.173 per day and a standard deviation of 0.066 per day having a coefficient of variation of 38 percent. This statistical model for K_1 is adapted for simulation studies using the Monte Carlo method.

Effect of Sludge Deposits

One of the greatest difficulties in stream surveys for estimating assimilative capacities is to evaluate the values of K_3, K_b, and D_b individually. To overcome the difficulty of estimating K_3 and K_b values Velz and Gannon (1962), Gannon (1963), O'Connor (1967), and others advocate the use of the term K_r which defines the coefficient for the BOD removal rate due to the lumped effects of K_1, K_3, K_b, and other unaccounted causes. In the absence of significant sludge deposits and fixed vegetation, the values of the rate coefficients K_3 and K_b can be taken as zero leaving only the oxidation of organic matter as the significant operative mechanism responsible for the removal of BOD from the system. Bottom samples collected during the 1957 study of the Ohio River-Cincinnati Pool (USPHS, 1960) revealed that there were no significant sludge deposits in the reaches investigated to warrant recognition of this as an important factor in the oxygen balance studies. This is further substantiated by the experience of several investigators (Nejedly, 1966; Thomas, 1948; Velz, 1958), that the problem of sludge deposits is greatly alleviated if settleable solids are removed from the raw waste prior to discharge into rivers. Since all the wastes emanating from Cincinnati, Ohio received primary treatment, and since the survey report indicates that the sludge deposits were insignificant, all the mechanisms of BOD and DO removal governed by sludge deposits can be treated as insignificant and consequently K_3 can be taken as zero.

In deep rivers such as the Ohio River, BOD removal due to extraction and accumulation of fixed plants is insignificant (Gannon, 1963; Heukelekian, 1967). It is relevant here to quote what Gannon (1963) has to say on river BOD removal mechanisms:

> Not all rivers have high BOD removal and many have rates which closely parallel the rates determined in the laboratory bottle experiments. This appears to be particularly true for the large rivers, where there is relatively less contact with river bottom and sides and where there is no gross dispersed type of growth.

Thus, it is seen that the only significant mechanism of any consequence affecting the BOD removal in the Ohio River-Cincinnati Pool area is that due to the biological oxidation of organic matter. Also consequent to the absence of significant sludge deposit, the benthal demand for oxygen is negligible.

Reaeration Coefficient

Churchill *et al.* (1962) suggest that when the regression equation, Eq. 23, is applied to polluted streams, the basic reaeration rates should be modified on a percentage basis as indicated by relative reaeration rates determined in the laboratory by experiments for the polluted water and for water samples obtained upstream of waste discharge. It is generally considered that the constituents in waste waters tend to reduce reaeration rates. Downing *et al.* (1957), and Downing and Truesdale (1955) conducted experiments to determine the effects of several contaminants and mixtures of contaminants on the exchange coefficient for oxygen in water agitated at different rates in laboratory absorption vessels. They concluded that household detergents reduced exchange coefficients in clean water by amounts which depended on their initial concentration and rate of agitation. They further concluded that the effects of settled sewage in the absence of any added anionic detergent were lower than when these materials were present. The reported percent reduction in K_2 values, under laboratory experimental conditions due to municipal wastes varies from 10 to 30 depending on the type and concentration of the constituents and the rate of agitation (O'Connor, 1958; Downing et al., 1957; Poon and Campbell, 1967). Poon and Campbell (1967) found that at low concentrations of suspended solids in tap water, the transfer rates were enhanced to the extent of 10 to 30 percent. Since the characteristics and concentrations of waste discharges are subject to considerable fluctuations, no attempt has been made to assign a numerical value for the possible effect of the pollutants on the mean river reaeration coefficient as predicted by the regression equation used in this study. Since the reaeration coefficient is treated as a random variable comprising of trend and random components, the error due to neglecting the effects of waste constituents which have compensating effects on the reaeration coefficient is considered negligible.

The values of velocity of flow, and depth used for determining the trend components of K_2 are shown in Table 11, for six different cases in which observations were made at two different sections of the river reach under consideration. In these cases, the downstream samples were obtained presumably after a time lag equal to the flow-through time from the upstream section of the reach with the result that the same body of water had been sampled as it flowed down the river. The percent deviation of actual K_2 value from the predicted value was taken to be normally distributed with mean zero and standard deviation of 36.8 percent. The order of magnitude of the residual error distribution of K_2 values applicable to the Ohio River study has been assumed to be the same as that for the Tennessee Valley rivers.

TABLE 11

VALUES OF PARAMETERS USED IN THE SIMULATION MODEL

Details of Initial Sample Collection			Average Velocity, fps	Average Depth, ft	Maximum Rate of Oxygen Addition Due to Photosynthesis, mg/(l)(day)	Average Rate of Respiration, mg/(l)(day)
River Mileage	Date	Time				
474.6	09-11-57	1105	0.26	19.6	3.95	1.80
474.6	09-11-57	1855	0.29	19.6	3.95	1.80
475.1	09-11-57	1930	0.25	19.6	3.95	1.80
474.6	09-12-57	0045	0.29	19.6	3.95	1.80
475.1	10-10-57	2315	0.34	19.3	2.28	0.98
476.2	10-23-57	0905	0.36	19.3	2.73	1.19

Photosynthesis and Respiration

Values for these parameters computed from the dark and light bottle observations reported for the Ohio River (USPHS, 1960) are shown in Table 11. Since photosynthesis and respiration of phytoplankton are greatly dependent on temperature and sunlight intensity, it is likely that these parameters, over a long period of observation will tend to show a trend in their values. It is more realistic to consider these parameters as variables with trend and random components since the factors which affect these parameters fluctuate having a trend with diurnal variations. Since the scope of the Ohio River survey light and dark bottle studies was limited to 12 days spread over two months, there is not enough information to discern these trends. As the maximum values for photosynthetic oxygen addition are assumed to occur around mid-day, the number of such observations made during the survey is inadequate to formulate and test any statistical model to characterize these parameters. Only arithmetic average values of these parameters obtained from the observations for different periods are used in this study. It is conceptually feasible to consider these two parameters also as random variables provided enough data could be collected to establish the trend and random components.

SAMPLE SIZE IN SIMULATION STUDIES OF THE OHIO RIVER

It was indicated earlier that the required sample size in simulation studies could be determined using Eq. 46, if the desired precision and confidence levels are known. Flagle (1960) suggests values of 10 percent and 95 percent for error and confidence levels respectively. Referring to Table 1 of Massey (1951) for the critical values of $d_{\alpha}(n)$ of the maximum absolute difference between sample and population cumulative distributions, the critical value of 'd' statistic in the Kolmogorov-Smirnov test for 95 percent confidence level is given as

$$d_{0.05} = \frac{1.36}{\sqrt{n}}$$

Thus, substituting the values for c_{α} and β, being 1.36 and 10 percent respectively in Eq. 46, and solving for n, the required sample size is indicated as 185 or approximately 200.

Hence in order to have confidence level of 95 percent that the information on required distribution is not different from the actual value by more than 10 percent, the sample size is to be 200.

RESULTS OF THE MONTE CARLO METHOD APPLIED TO THE OHIO RIVER

A flow diagram for the computer simulation studies using the Monte Carlo method following the general procedure enumerated in Chapter V is shown in Appendix C. Two separate sets, each having 200 random numbers with uniform distribution (0,1) were generaged using a sub-routine available with the University of Illinois Digital Computer Laboratory. The first set of random numbers was transformed to correspond to K_1 values with a mean value of 0.173 per day and standard deviation of 0.066 per day. The second set of random numbers was transformed to correspond to the random variations in percent error of predicted K_2 values with a mean value of zero and standard deviation of 38.6 percent.

It is likely that some extremely and unreasonably high or low values will be generated by the Monte Carlo method and it becomes necessary to ignore such values (Montgomery and Lynn, 1964) or these have to be corrected on a reasonable basis (Ramaseshan, 1964). In this study the transformed variables with values beyond twice the deviation from the stipulated mean were assigned mean values. The generated random variates were tested for goodness of fit with the assumed distributions using the Kolmogorov-Smirnov test. The distributions of the generated values were found to satisfy the test.

With known conditions for velocity and depth of flow, the trend component of K_2 could be evaluated using Eq. 31. Knowing the mean K_2 values for the given conditions of flow, the random variations in K_2 could be computed from the generated random variations in percent error. These generated random values for the variations in K_2 values were added algebraically to the mean value to obtain the random values of K_2.

The river system for determining the assimilative capacity was simulated using Eqs. 53 and 54 in which K_3, K_b, and D_b were taken as zero for reasons discussed earlier. Generated values of K_1, K_2, along with mean values for P_m and R indicated in Table 11 were used in determining the response of the system. The values for DO deficit predicted by Monte Carlo techniques were verified with observed values in six different cases in which the same body of river water had been sampled at two different sections downstream of all the major waste discharges emanating from Cincinnati, Ohio. In each of these cases, the process was simulated with known initial conditions and the state of dissolved oxygen in terms of oxygen deficit was predicted. The initial conditions of ultimate BOD, dissolved oxygen, temperature, and the final conditions of dissolved oxygen and temperature along with the flow-through time between the two sections are presented in Table 12, for each of the six cases considered.

Two hundred values for dissolved oxygen deficit values were generated for each case study solving the differential equations using a subroutine in the University of Illinois Digital Computer Laboratory. The subroutine employs fourth order Runge-Kutta method for solving differential equations, the details of which can be found in standard textbooks on numerical analysis (Fox, 1962; McCracken and Dorn, 1964). Utilizing the generated values of DO deficit, the frequency distribution of probable values was computed and these are shown in Table 13, and plotted in Figures 14 through 19. The <u>expected</u> and <u>most probable</u> values of DO deficits for each of these cases are shown in Table 14. In order to evaluate and compare the results obtained by Monte Carlo techniques, predicted values of DO deficits were computed for each case using equations postulated by Streeter and Phelps (1925), and Camp (1963). Average value of 0.173 per day for K_1 and the value obtained from the regression equation, Eq. 31, for K_2 using the hydraulic characteristics reported in Table 11 were used in these formulae. These results are presented in Table 14. Percent error in predicting DO deficit was computed by multiplying 100 with the fraction obtained by dividing the absolute difference between actual and predicted values of DO deficit with the actual deficit.

TABLE 12

INITIAL AND FINAL CONDITIONS OF THE OHIO RIVER SAMPLES FOR
SIX DIFFERENT CASES USED TO VERIFY MONTE CARLO PREDICTIONS

Case Study No.	Sample Collection: River Mileage	Sample Collection: Date	Sample Collection: Time	Temp. °C	Ultimate First Stage BOD, mg/1	DO, mg/1	Flow Time, days
1	474.6	09-11-57	1105	24	9.8	2.76	0.70
	477.5	09-12-57	0400	24		1.88	
2	474.6	09-11-57	1855	24	8.5	2.80	0.34
	476.2	09-12-57	0315	24		2.33	
3	475.1	09-11-57	1930	24	8.4	3.53	0.59
	477.5	09-12-57	0940	24		1.37	
4	474.6	09-12-57	0045	24	10.0	3.09	0.35
	476.2	09-12-57	0900	24		1.62	
5	475.1	10-09-57	2315	19	13.0	8.36	0.73
	479.1	10-10-57	1645	19		7.49	
6	476.2	10-23-57	0905	16	11.2	8.26	0.49
	479.1	10-23-57	2100	16		7.62	

TABLE 13

FREQUENCY DISTRIBUTION OF PROBABLE VALUES OF DO DEFICITS

Case Study No. 1		Case Study No. 2		Case Study No. 3	
Class Interval DO Deficit, mg/l	Frequency	Class Interval DO Deficit, mg/l	Frequency	Class Interval DO Deficit, mg/l	Frequency
6.26 - 6.50	0.020	6.26 - 6.50	0.030	5.76 - 6.00	0.030
6.51 - 6.75	0.055	6.51 - 6.75	0.190	6.01 - 6.25	0.110
6.76 - 7.00	0.100	6.76 - 7.00	0.585	6.26 - 6.50	0.185
7.01 - 7.25	0.150	7.01 - 7.25	0.195	6.51 - 6.75	0.430
7.26 - 7.50	0.365			6.76 - 7.00	0.160
7.51 - 7.75	0.160			7.01 - 7.25	0.085
7.76 - 8.00	0.100				
8.01 - 8.25	0.050				

TABLE 13 (Continued)

FREQUENCY DISTRIBUTION OF PROBABLE VALUES OF DO DEFICITS

Case Study No. 4		Case Study No. 5		Case Study No. 6	
Class Interval DO Deficit, mg/1	Frequency	Class Interval DO Deficit, mg/1	Frequency	Class Interval DO Deficit, mg/1	Frequency
5.76 - 6.00	0.045	1.25 - 1.50	0.055	1.76 - 2.00	0.065
6.01 - 6.25	0.130	1.51 - 1.75	0.075	2.01 - 2.25	0.200
6.26 - 6.50	0.460	1.76 - 2.00	0.115	2.26 - 2.50	0.490
6.51 - 6.75	0.240	2.01 - 2.25	0.340	2.51 - 2.75	0.195
6.76 - 7.00	0.140	2.26 - 2.50	0.200	2.76 - 3.00	0.050
		2.51 - 2.75	0.120		
		2.76 - 3.00	0.075		
		3.01 - 3.25	0.020		

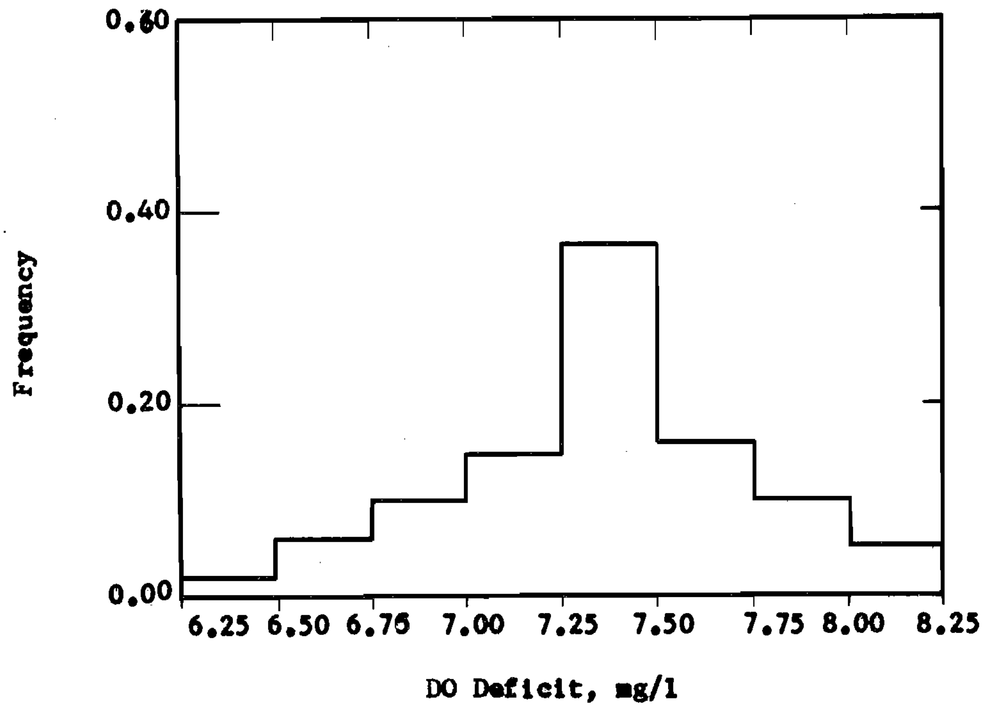

FIGURE 14. FREQUENCY DISTRIBUTION OF DO DEFICIT IN CASE STUDY NO. 1

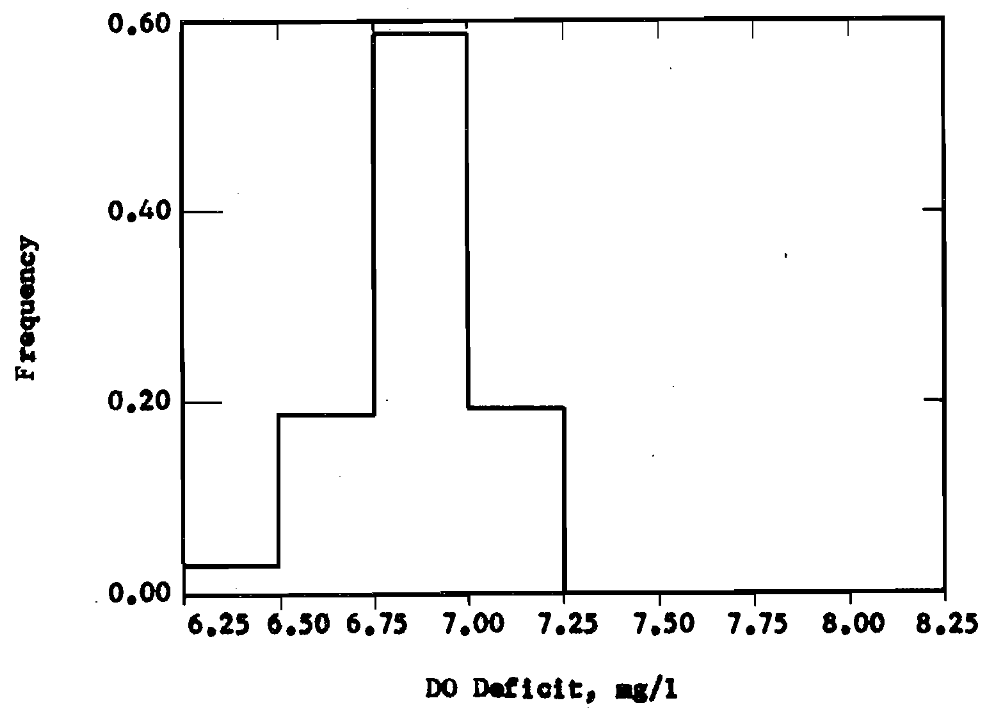

FIGURE 15. FREQUENCY DISTRIBUTION OF DO DEFICIT IN CASE STUDY NO. 2

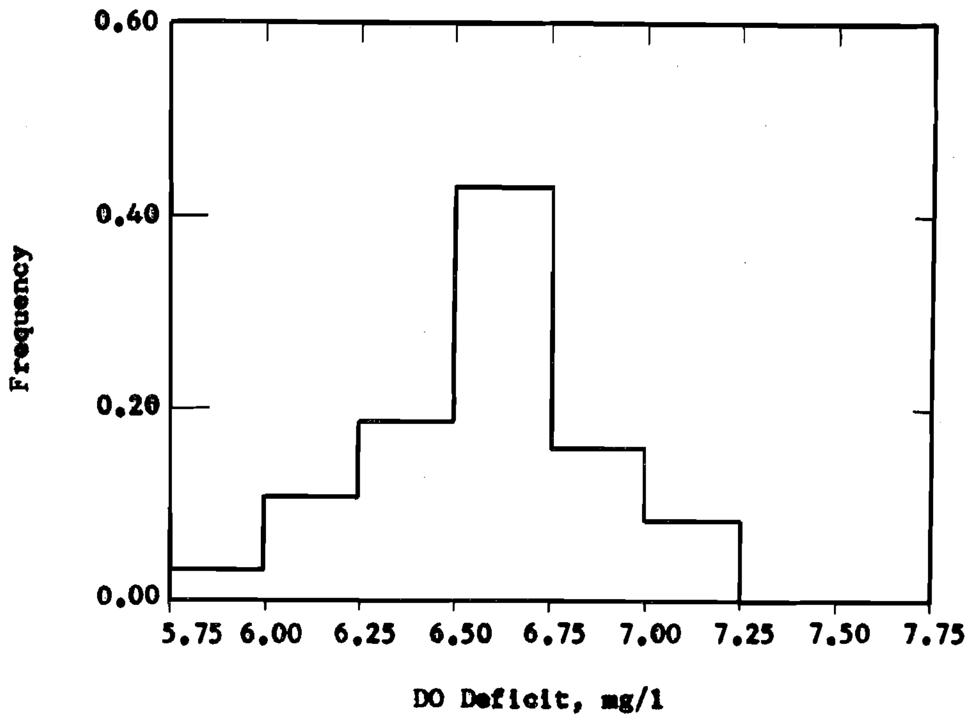

FIGURE 16. FREQUENCY DISTRIBUTION OF DO DEFICIT
IN CASE STUDY NO. 3

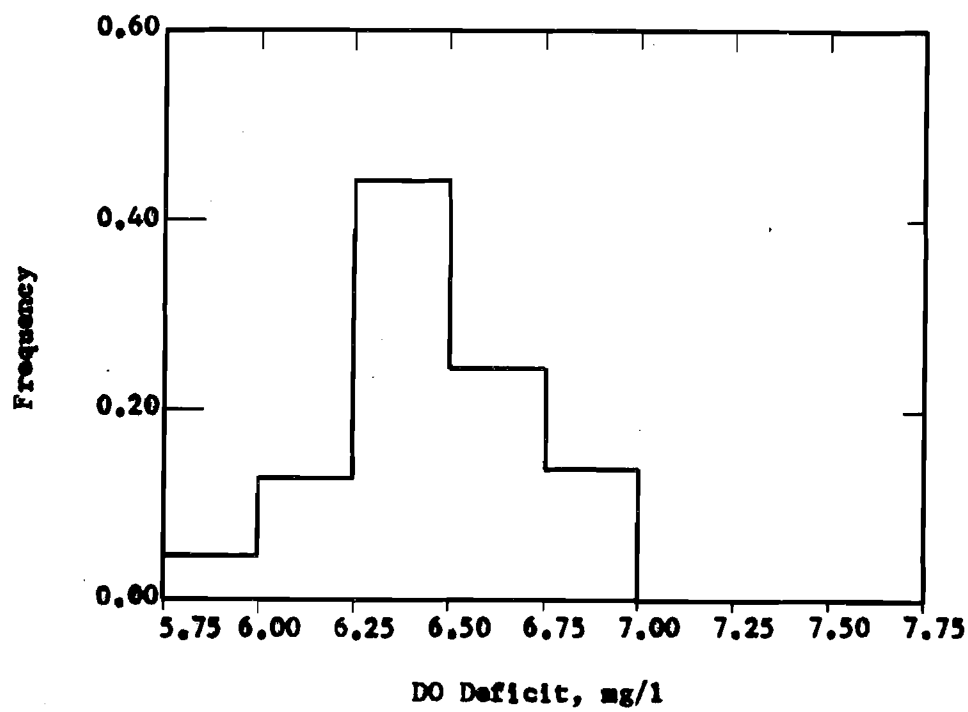

FIGURE 17. FREQUENCY DISTRIBUTION OF DO DEFICIT
IN CASE STUDY NO. 4

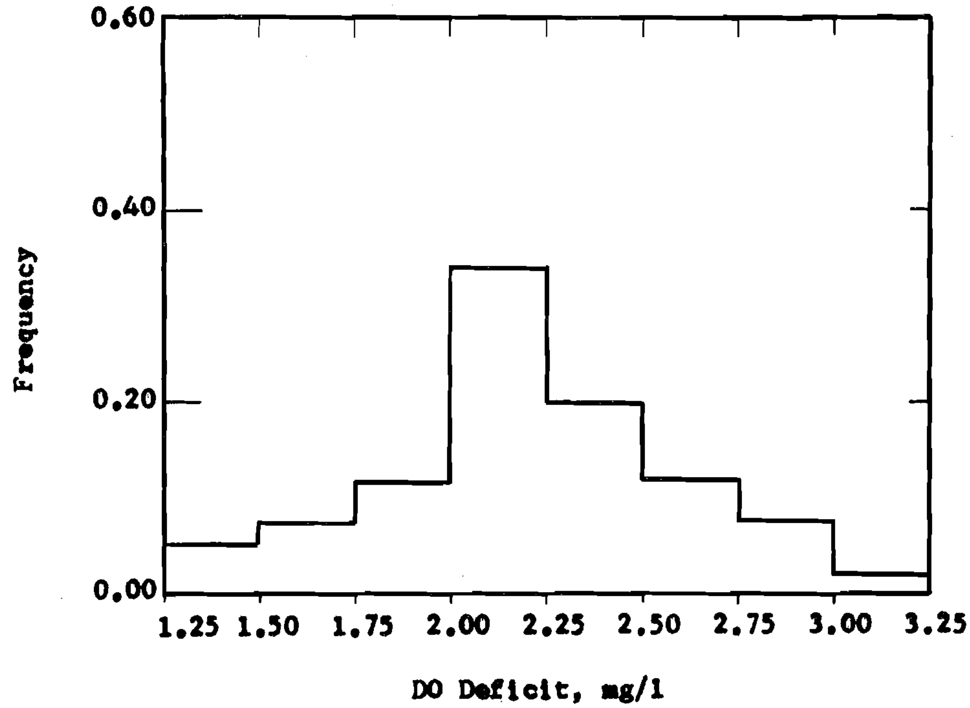

FIGURE 18. FREQUENCY DISTRIBUTION OF DO DEFICIT IN CASE STUDY NO. 5

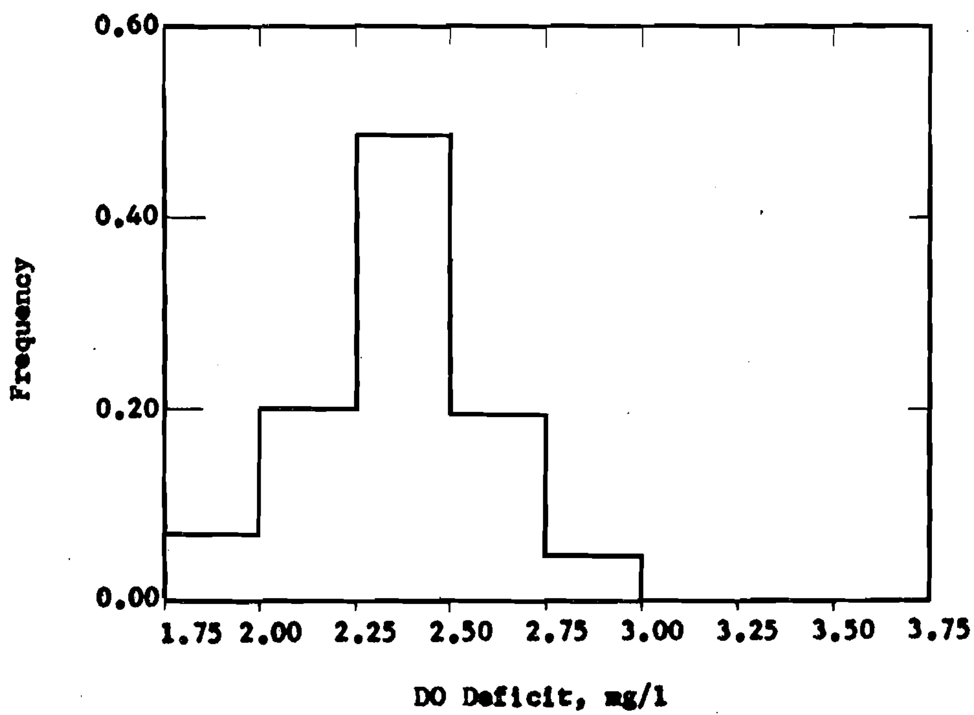

FIGURE 19. FREQUENCY DISTRIBUTION OF DO DEFICIT IN CASE STUDY NO. 6

TABLE 14

ACTUAL AND PREDICTED DO DEFICITS

Case Study No.	DO Deficit, mg/l					Percent Error			
				Probabilistic Model				Probabilistic Model	
	Actual Value	Streeter-Phelps' Eq.	Camp's Eq.	Expected Value	Most Probable Value	Streeter-Phelps' Eq.	Camp's Eq.	Expected Value	Most Probable Value
1	6.62	6.84	6.08	7.34	7.38	3.3	8.1	10.9	11.5
2	6.17	6.20	5.77	6.86	6.88	0.5	6.5	11.1	11.5
3	7.13	5.44	4.66	6.58	6.63	23.7	34.7	7.7	7.0
4	6.88	6.00	5.57	6.45	6.38	12.9	19.0	6.3	7.4
5	1.91	2.55	2.20	2.21	2.13	33.5	15.3	15.7	11.5
6	2.38	2.62	2.21	2.37	2.38	10.1	7.5	0.4	0.0

VII. DISCUSSION

The significance and importance of considering the variations in K_1 and K_2 were brought out in Chapter III. In the carefully selected river reaches of the Tennessee Valley rivers, the atmospheric reaeration was found to vary significantly in every reach considered, even though each set of observations was made under constant river flow conditions. The observed range of values for K_2 was considerably more for shallow and high velocity flows than for deep and low velocity flows. As pointed out earlier, in one set of experiments in the Holston River, K_2 had a range of values from 0.10 to 1.18 per day with an arithmetic average of 0.63 per day. All the mathematical models developed for predicting the reaeration rate coefficient attempt to estimate only the average value for K_2. Efforts so far made to evaluate the stream assimilative capacities fail to take into account the variations in the actual values of K_2 from the predicted values. These variations are considerable and lead to significant errors in predicting the DO response, if omitted.

Attempts have been made to relate the reaeration coefficient to such hydraulic characteristics as velocity and depth of flow, energy slope, surface renewal rates, etc. These were not found to explain adequately the variations in the predicted values for K_2. Inclusion of the variables, such as molecular diffusion, dispersion coefficient, surface tension, etc., did not significantly improve the predictions for reaeration coefficients (TVA, 1962). Variations in the values for reaeration coefficient may be attributed to the errors in estimation of the parameters involved in the predictor equations, simplification in the assumptions for developing the equations and other uncontrollable and unaccounted causes such as wind effects, turbulence, etc. As these factors are not quantifiable in a deterministic sense, the combined effects have to be expressed in terms of probability measure. For the Tennessee Valley rivers, the variations in reaeration coefficients have been found to be adequately defined by Gaussian distribution.

Though the deoxygenation and reaeration coefficients were the only two parameters treated as random variables in this study, the probabilistic concepts should be extended to other factors such as photosynthesis, respiration, biological extraction of BOD, etc., which are subject to random fluctuations. Also the covariance of these factors needs to be given consideration. Attempts have been initiated at the University of Illinois to study the possible effects of different rates in deoxygenation on atmospheric reaeration capabilities in laboratory channels. Considerable additional research is needed in this area.

Monte Carlo techniques applied to the problem of determining the DO response of streams receiving organic waste loads appear to yield satisfactory results. The results of the analysis of the Ohio River survey data presented in Table 14 indicate that there is considerable difference between the DO deficits actually observed and those predicted by the deterministic equations. Application of the Streeter-Phelps equation in which the effects of photosynthesis and respiration are omitted and in which the reaction coefficients are taken as constants in a river reach, results in prediction errors ranging from 0.5 to 33.5 percent. Use of Camp's equation (1963) which accounts for an average value of photosynthesis also results in prediction errors ranging from 6.5 to 34.7 perdent. The use of an average value for photosynthesis in Camp's model irrespective of daytime or nighttime flow conditions is unrealistic since no photosynthesis can be expected to occur during nighttime. As the characteristics of the waste emanating even from a single source vary from time to time and the type and distribution of microflora stabilizing the waste organic matter are bound to change in the natural environment, it is realistic to consider all the possible variations in parameters characterizing these changes instead of assuming them to be time invariant.

The percent error involved in predicting DO deficits using the probabilistic model range from 0.4 to 15.7 when the predictions are based on expected values and range from 0.0 to 11.5 when based on most probable values. It is seen that under different conditions, the values of DO deficit predicted by the probabilistic model, particularly the most probable values, tend to be closer to the observed values all the time than the values predicted by the deterministic equations. The concept of most probable value is not unfamiliar in sanitary engineering practice, since the enumeration of coliform organisms in water samples involves this concept.

The average value for the percent errors in DO deficits predicted by the Streeter-Phelps equation for the six cases investigated in this study is 14.0 and the corresponding value for the Camp equation is 15.2. The average values in percent errors using the probabilistic model are 8.7 and 8.2 respectively for the expected value and most probable value cases. Thus it is seen that the use of most probable value predicted by the probabilistic model is likely to give more reliable information on the DO response than the deterministic equations considered in this study. However the Streeter-Phelps model has predicted the DO deficit within one percent error when the initial and final observations were carried out at night (Case Study No. 2) and the error in prediction using Camp's equation for this case is 6.5 percent which is the best value obtained employing Camp's model.

The errors inherent in the predictions using probabilistic model should be considered as being within practical limits, since several approximations and uncertainties creep in the evaluation of various factors which affect the DO-BOD relationship in streams. The estimation of time of flow between two sections in a river reach could at best be only approximate, so also the average values of depths and velocities of flow. Any errors in the assumption of oxygen saturation values at different temperatures are bound to be reflected in the final results of DO deficit predictions. Again there is the raging controversy about the adequacy of BOD bottle experiments to duplicate the oxidation process in the unrestricted natural environment. Sanitary engineers have to put up with this, being the only expedient method available at present to evaluate the biodegradability of organic matter.

The use of the regression equation, Eq. 31, developed from the data collected in Tennessee Valley rivers, for computing mean values of K_2 applicable to the Ohio River involves extrapolation to conditions other than those for which the equation was developed. The validity and usefulness of the equations developed by O'Connor and

Dobbins (1956) and Dobbins (1964) were not established beyond reasonable doubt. The discrepancies in observed and predicted values using these equations have been pointed out (Churchill *et al*., 1962; Thackston and Krenkel, 1965). The use of the regression equation developed for Tennessee Valley rivers, being the best available tool for estimating the reaeration coefficient independently, might be another source of error, since this involves extrapolation as indicated earlier.

The strength of the proposed method lies in the fact that in the face of several ambiguities, it predicts a range of values for DO deficits with associated probabilities instead of a single value as in deterministic equations. Thus it enables one to quantify the uncertainties in terms of probability measure and to consider the probability of river DO being equal to or less than certain concentrations under given waste and stream flow conditions.

Figure 20 shows the probable range of values for the oxygen sag profile in the Ohio River and the associated probabilities at a few selected sections for the initial conditions represented in Case Study No. 5. The histograms are drawn with class intervals of 0.25 mg/l. Class intervals smaller than 0.25 mg/l are considered unwarranted in view of the sensitivity of the DO determination methods. The profiles predicted by Camp's and Streeter-Phelps' equation are also shown in Figure 20. For short times of flow, the values of DO predicted by the two deterministic equations and the modal value of the probabilistic model are in close agreement within practical limits. As the time of flow increases, there is considerable deviation in the values predicted by these methods. The effect of the diurnal variations in the photosynthesis is seen to be significant during daytime flow conditions.

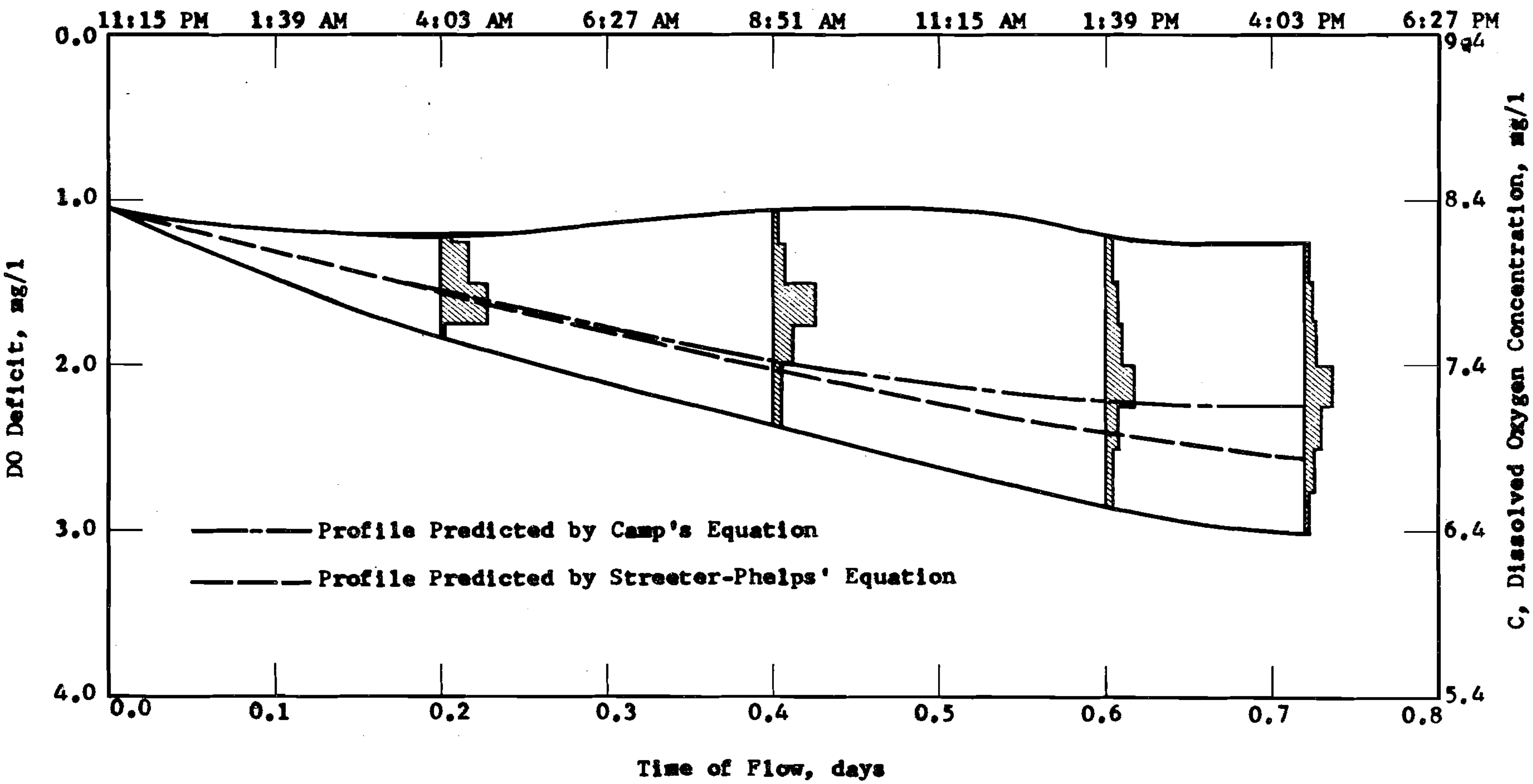

FIGURE 20. RANGE OF DO AND THE ASSOCIATED PROBABILITY AT DIFFERENT SECTIONS OF THE OHIO RIVER FOR THE CASE STUDY NO. 5

The probability distributions of DO at different sections of the river shown in Figure 20 pertain to the extent and intensity of exposure to the daylight. These factors depend on the time of initial observation at the upper end of the river reach considered.

In the case of the probabilistic model, it is not possible to consider critical dissolved oxygen conditions as in deterministic models. The objective in waste water treatment systems design and operation will then be to minimize the probability of the dissolved oxygen level in the stream falling below the stipulated value. This could be evaluated knowing the probability distributions of the dissolved oxygen concentrations at different sections of the river reaches for the known initial conditions.

The writer became aware at a late stage of the possibility of employing alternative methods of finding theoretically the probability distribution function for the DO deficit, when the probability distribution functions for K_1 and K_2 and their functional relationship with DO deficit are known. These alternatives are finding the distribution of a function of one or more random variables using <u>change of variable technique</u> or the <u>moment-generating function technique</u>. The analytical technique will yield a more precise information than the Monte Carlo method.

A need for taking into account on a rational basis the variations in the parameters affecting the DO-BOD relationship had always been felt and this is at least partly met in this study. The difficulties one faces in predicting DO responses in a river system are best summarized by quoting Kneese (1964):

> Much has been done to develop generalizations about the receiving water environment and to apply scientific principles to it. However precision is less and uncertainty greater in this area than in most fields of scientific and engineering forecasting. This emphasizes the importance of empirical checking of forecasts made upon the basis of highly simplified principles.

VIII. ENGINEERING SIGNIFICANCE

New methods are reported for evaluating the composite effects of combinations of waste water treatment, waste water flow regulation, stream flow regulation, optimal allocation of stream dissolved oxygen, etc. (Liebman and Lynn, 1966; Loucks and Lynn, 1966; Worley, 1963), for application in comprehensive programs to improve and maintain the quality of water in major water resources systems. Several new mathematical techniques useful in fusing engineering design, economic analysis and governmental planning have recently been developed to exploit the potentiality of the electronic computer. In these studies, the streams' self purification capacities and the allocation of stream dissolved oxygen resources among various sources of pollution play an important role and the reliability of the conclusions in these studies depends on the accuracies of dissolved oxygen predictions, to a smaller or larger extent. Thus the need for estimating the response of receiving streams to waste loads as accurately as possible becomes obvious.

Kneese (1964) advocates the need for considering the probabilistic character of "damage costs" which include the optimal combmination of water treatment costs and physical damages due to the waste discharged upstream of the point of use. The mathematical expectation of damages associated with a particular level of stream flow is obtained by multiplying the probability of each flow by the corresponding cost (treatment plus damage). If, in addition to the probability of occurrence of different stream discharges, the variations in the basic velocity coefficients which define the streams' self purification capacities are taken into account, a better estimate of the mathematical expectation of damages could be obtained. This in turn aids in management decisions aimed toward the best possible development of a water resources system.

There is a growing realization for the need of probabilistic oxygen standards in streams (Loucks, 1965; Thayer and Krutchkoff, 1966). In the earlier studies either the chance variations in stream flow (Loucks, 1965), or the variations in the states of BOD and DO (Thayer and Krutchkoff, 1966) were considered. In these cases the velocity coefficients K_1 and K_2 were treated as constants. Since the variations in K_1 and K_2 have been demonstrated to be significant, it is more realistic to consider the variations in these coefficients within their practical ranges. The concepts developed in this work can be extended to include variations in stream flows, thus obtaining more general information on the probability distribution of dissolved oxygen concentration in the stream. In view of the uncertainties involved in the stream self purification process, it is more realistic to introduce arguments of probability and predict, not how the system will certainly and invariably perform, but how it will perform a certain fraction of the time.

IX. SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE STUDY

SUMMARY OF THE STUDY

A digital computer model is used for defining the self purification process in the Ohio River-Cincinnati Pool reach. The model being a representation of the prototype incorporating those features of the prototype deemed to be important for the purpose at hand, the operation of the model in any time interval is characterized by the inputs (BOD) in accordance with the parameters of the process, yielding a sequence of outputs (DO deficit). If the system is affected or perturbed by random components implicit in the input, any single simulation run yields only a specific solution related to one set of conditions of the system. To determine the general relation of the output to the input and variables may require a large number of simulation runs. Monte Carlo techniques are combined with simulation analysis to remedy the lack of generality implicit in simulation solutions.

In applying Monte Carlo simulation techniques to the Ohio River-Cincinnati Pool reach, the mechanisms of BOD removal with concomitant removal of oxygen due to bacterial oxidation of organic matter, oxygen addition due to photosynthesis and atmospheric reaeration and algal respiration are considered. Other known mechanisms affecting the DO-BOD relationship such as removal or addition of BOD due to deposition or scour, oxygen demand due to benthal deposits, etc., are considered insignificant in the oxygen balance studies for the particular case investigated. Oxygen addition due to photosynthesis is treated as a periodic function having zero values at sunrise and sunset and a maximum value at mid-day. Only arithmetic average values for algal respiration and maximum rate of oxygen addition due to photosynthesis are considered. The reaction coefficients K_1 and K_2 are treated as random variables. A hypothetical case using Streeter-Phelps' formulation is used to establish the need for considering the variations in K_1 and K_2.

Two hundred values for K_1 and K_2 are generated by Monte Carlo methods. Using these generated values of the reaction coefficients and average values for other parameters involved in the process, 200 values for the dissolved oxygen deficit are generated, giving a probable range of values for the dissolved oxygen deficit with associated probabilities. The results of the dissolved oxygen deficits predicted by the probabilistic model are compared with the observed values. The predictions for dissolved oxygen deficits based on the most probable values are found to be in close agreement all the time with the observed values.

To summarize the procedure involved in the use of Monte Carlo techniques for determining the self purification capacities of a stream under steady state flow conditions, the first step is to define all the mechanisms which affect BOD and DO, and to formulate a mathematical model describing the DO response. Probability distributions for the significant parameters which are found to vary considerably in the model are determined based on actual observations for these parameters. The number of simulation runs required to predict the DO response under the known initial conditions is dictated by the precision and the confidence with which it is to be predicted. The required number of values for the parameters which are found to vary with known probability distributions are obtained by suitable transformation techniques from generated random numbers. These generated values, which should conform to the probability distributions of the historical samples, are then used in the simulation model to obtain probable range of values for the DO response with its associated probability distribution.

CONCLUSIONS

On the basis of this study, the following conclusions can be drawn:

1. The variations in the values of deoxygenation and reaeration coefficients within their practical ranges have significant effect in the prediction of the state of dissolved oxygen in streams receiving organic waste loads. Consequently there is little justification in considering only the average values for these coefficients instead of taking into account all the probable values in any given reach.

2. The error in prediction of dissolved oxygen due to the variations in reaction coefficients increases with increase in temperature.

3. The values for deoxygenation coefficient for river samples obtained downstream of all the major waste outfalls in Cincinnati, Ohio are found to vary at random. This is probably due to the random changes in the characteristics of the waste and the changes

in the type and distributions of microorganisms responsible for the stabilization of organic matter. The variations in deoxygenation coefficient values are adequately represented by Gaussian distribution with a mean value of 0.173 per day, a standard deviation of 0.066 per day and a coefficient of variation of 38 percent.

4. The variations in percent error in the prediction of reaeration coefficient values for Tennessee Valley rivers using regression equation with mean depth and velocity of flow as independent variables follow normal distribution law with a mean of zero and a standard deviation of 36.8 percent.

5. The most probable values for the dissolved oxygen deficits predicted by the probabilistic model using Monte Carlo simulation techniques are found to be better estimates than the values predicted by the conventional deterministic approaches.

SUGGESTIONS FOR FUTURE STUDY

Based on the results of this study and the understanding of the probabilistic process of the DO-BOD relationship in streams, the following suggestions for future research are proposed:

1. The applicability of Monte Carlo techniques has to be verified for shallow streams where other mechanisms, in addition to the ones considered in this study, affecting the DO-BOD relationship are significant.

2. The interdependence of the reaction coefficients K_1 and K_2 need further investigation. If they are not found completely independent, as have been assumed in this study, a suitable joint density function has to be developed and used in the probabilistic model.

3. The variations in other parameters like oxygen addition due to photosynthesis, algal respiration, etc., need to be studied and accounted for in the probabilistic model.

4. The concept of a simulation technique using Monte Carlo methods may be extended to include variable initial conditions arising from temporal variations in river flow, waste flow and waste strength.

5. Research in the area of optimal design of waste treatment facilities, optimal allocation of stream dissolved oxygen, etc., should include the probabilistic variations in the parameters affecting streams' waste assimilative capacities.

6. The probabilistic model proposed in this study has to be solved analytically to determine the probability distribution function for the DO deficit employing either the change of variable technique or the moment-generating function technique.

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APPENDIX A

NOTATION

A_i	Coefficients in the linear regression equation.
C_α	Constant defining the critical value of 'd' statistic in Kolmogorov-Smirnov test for samples of size greater than 35.
C	Dissolved oxygen concentration, mg/l, in stream at any time.
C_s	Saturation DO concentration, mg/l, at any given temperature.
c.d.f.	Cumulative distribution function.
D	DO deficit, mg/l, at any given temperature and time of flow.
D_a	Initial DO deficit, mg/l, in a river reach.
D_b	Oxygen demand, mg/(l)(day), due to benthal deposits.
D_c	Critical DO deficit, mg/l, in a stream.
D_m	Coefficient of molecular diffusion, L^2/T.
$d_\alpha(n)$	Critical value of 'd' statistic in Kolmogorov-Smirnov test for a sample size of n observations at a significance level of α.
$E(\cdot)$	Expected value of a function.
e_i	Expected number of samples in category i in chi-square test.
F(x)	Cumulative frequency distribution.
f	Degree of freedom in chi-square test.
f(x)	Probability density function.
g	Number of population parameters estimated from the sample.
H	Mean depth, feet.
K_1	Rate coefficient, day^{-1}, defining BOD removal with concomitant DO removal process (base e).
K_2	Rate coefficient, day^{-1}, defining atmospheric reaeration process (base e).
K_3	Rate coefficient, day^{-1}, defining addition or removal of BOD process due to scour or deposition.
K_b	Rate coefficient, day^{-1}, defining BOD removal due to biological extraction and accumulation.

K_r — Rate coefficient, day^{-1}, defining overall removal rate of BOD in a river system.

k — Number of observations less than or equal to a given value x.

L — First stage ultimate BOD, mg/1, remaining to be satisfied.

La — Initial first stage ultimate BOD, mg/1, in a river reach.

n — Total number of observations made in a series of observations.

n_i — Number of samples observed in category i of chi-square test.

$P\{\cdot\}$ — Probability of observing a value less than or equal to a stated value.

P_i — Number of phases of length i in "turning points" test.

P_m — Maximum rate of oxygen addition, mg/(1)(day), due to photosynthesis.

P_t, — Rate of oxygen addition, mg/(1)(day), at any given time.

p — Total number of peaks in "turning points" test and period of oscillation in a periodic sine function.

p.d.f. — Probability density function.

R — Average rate of algal respiration, mg/(1)(day), and total number of runs in the "runs up and down" test.

R_k — Number of runs of length k or more in the "runs up and down" test.

$R_{0.1,2,...,p}$ — Multiple correlation coefficient.

r — Total number of categories in chi-square test.

r_i — Number of runs of length i in the "runs up and down" test.

S — Slope of river channel.

S_E^2 — Sum of squared errors in regression analysis.

$S_n(x)$ — Observed cumulative step function of a sample of observations.

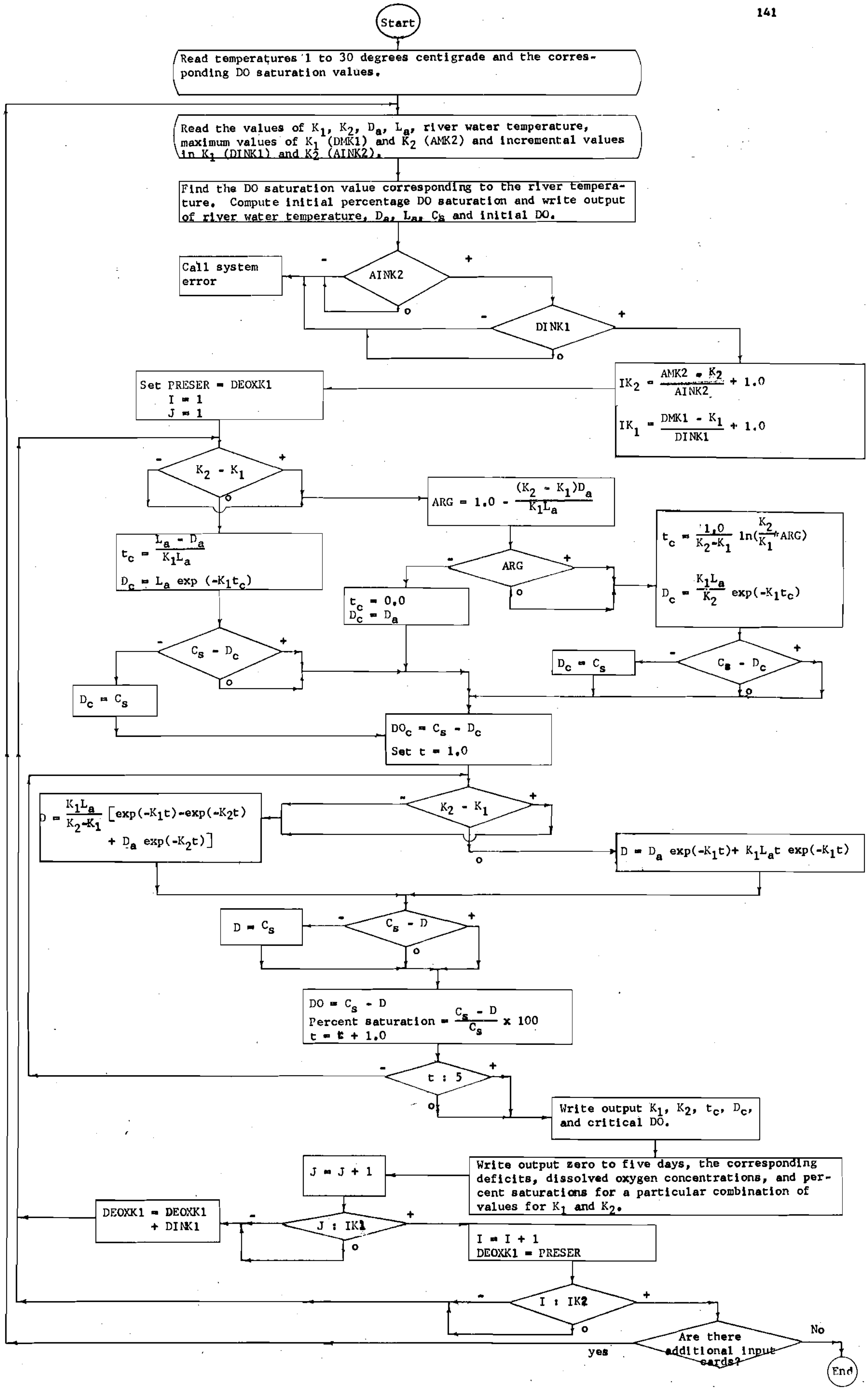
T — River temperature, degrees centigrade.

t — Time of flow, days.

t'	Time of flow, in days, of a mass of water since sunrise after that mass of water has entered the head end of a river reach considered.
t_c	Flow through time, days, for critical section in a river.
V	Mean velocity of flow, ft/sec, in a river reach.
$V(\cdot)$	Variance of a function.
X	Random variable.
x	Any particular value a random variable X assumes.
Z_i	Independent variables in regression analysis.
Z_o	Dependent variable value predicted in linear regression equation.
Z_O'	Actual value of dependent variable.
α	Significance level.
β	Error level in predicted values.
Δ	Incremental value in the variables.
θ	Temperature coefficient defining deoxygenation rates at different temperatures.
Φ	Temperature coefficient defining reaeration rates at different temperatures.
σ	Variance in population.
χ^2	Chi-square statistic.

APPENDIX B

FLOW DIAGRAM FOR OXYGEN SAG COMPUTATIONS; SENSITIVITY ANALYSIS



Start
Read temperatures 1 to 30 degrees centigrade and the corresponding DO saturation values.
Read the values of K_1, K_2, D_a, L_a, river water temperature, maximum values of K_1 (DMK1) and K_2 (AMK2) and incremental values in K_1 (DINK1) and K_2 (AINK2).
Find the DO saturation value corresponding to the river temperature. Compute initial percentage DO saturation and write output of river water temperature, D_a, L_a, C_s and initial DO.
Call system error
AINK2
DINK1
$IK_2 = \frac{AMK2 - K_2}{AINK2} + 1.0$
$IK_1 = \frac{DMK1 - K_1}{DINK1} + 1.0$
Set PRESER = DEOXK1
I = 1
J = 1
$K_2 - K_1$
$ARG = 1.0 - \frac{(K_2 - K_1)D_a}{K_1 L_a}$
$t_c = \frac{L_a - D_a}{K_1 L_a}$
$D_c = L_a \exp(-K_1 t_c)$
ARG
$t_c = 0.0$
$D_c = D_a$
$t_c = \frac{1.0}{K_2 - K_1} \ln(\frac{K_2}{K_1} * ARG)$
$D_c = \frac{K_1 L_a}{K_2} \exp(-K_1 t_c)$
$C_s - D_c$
$D_c = C_s$
$DO_c = C_s - D_c$
Set t = 1.0
$D = \frac{K_1 L_a}{K_2 - K_1} [\exp(-K_1 t) - \exp(-K_2 t) + D_a \exp(-K_2 t)]$
$K_2 - K_1$
$D = D_a \exp(-K_1 t) + K_1 L_a t \exp(-K_1 t)$
$D = C_s$
$C_s - D$
$DO = C_s - D$
Percent saturation = $\frac{C_s - D}{C_s} \times 100$
t = t + 1.0
t : 5
Write output K_1, K_2, t_c, D_c, and critical DO.
J = J + 1
Write output zero to five days, the corresponding deficits, dissolved oxygen concentrations, and percent saturations for a particular combination of values for K_1 and K_2.
DEOXK1 = DEOXK1 + DINK1
J : IK1
I = I + 1
DEOXK1 = PRESER
I : IK2
Are there additional input cards?
yes
No
End

APPENDIX C

FLOW DIAGRAM FOR SOLVING PROBABILISTIC MODEL FOR DO-BOD RELATIONSHIP

Start

B

Read input data; D_a, L_a, V, H, T, R, time of initial observation and other relevant parameters.

Generate a sequence of uniform (0,1) random numbers and evaluate the corresponding K_1 values. Correct the unreasonably high or low values of K_1.

Perform Kolmogorov-Smirnov test on the generated values of K_1.

Is the generated sample significantly different from the historical sample?

Yes

No

Compute the value of trend component of K_2 using regression equation.

Generate a sequence of uniform (0,1) random numbers and evaluate the corresponding random components of K_2 values. Correct the unreasonably high or low values for the random components of K_2.

Perform Kolmogorov-Smirnov test on the generated values for the random components of K_2.

Is the generated sample significantly different from the assumed distribution?

Yes

No

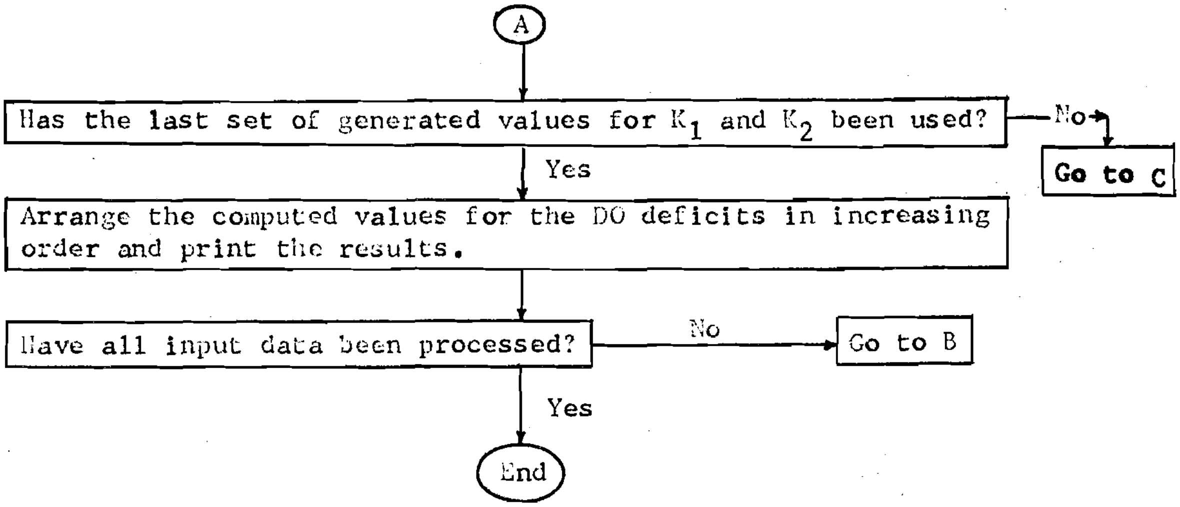
Develop random samples of K_2 values by adding the trend component to the random components of K_2.

Set up necessary initial conditions for solving the differential equations using the sub-routine for fourth order Runge-Kutta method.

C

Use a set of generated values for K_1 and K_2, apply temperature corrections to them. Compute oxygen deficit for the known initial conditions using appropriate values for other parameters in the process.

A


A
Has the last set of generated values for K_1 and K_2 been used?
No
Go to C
Yes
Arrange the computed values for the DO deficits in increasing order and print the results.
Have all input data been processed?
No
Go to B
Yes
End

APPENDIX D

BOD PROGRESSION DATA FOR THE OHIO RIVER SAMPLES (1957 SURVEY)

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/l						K_1, per Day	Ultimate First Stage BOD, mg/l
474.6	9/11	1105	0.99	1.86	2.95	4.85	6.82	8.86	0.280	9.8
			2.42	4.19	5.60	7.02	8.25	9.23		
	9/11	1855	0.85	1.52	2.60	4.51	6.48	8.52	0.284	8.5
			2.01	3.18	4.39	6.06	6.97	8.02		
	9/12	0045	1.02	2.21	3.12	4.91	7.06	9.38	0.240	10.0
			2.49	4.21	5.24	6.63	8.08	9.11		
	9/12	0710	0.95	1.96	2.88	4.83	6.81	9.12	0.277	9.9
			2.44	4.35	5.40	7.12	8.17	9.38		
	9/12	1900	0.68	1.68	2.59	4.89	6.53	8.84	0.254	9.8
			1.73	3.55	4.84	6.58	8.03	8.93		
	9/12	0210	0.96	1.88	2.86	4.81	6.88	8.92	0.239	10.4
			2.35	3.91	5.18	6.80	8.26	9.38		
475.1	9/11	0730	0.97	1.84	2.93	4.85	6.80	8.84	0.275	9.9
			2.62	4.01	5.54	7.05	8.25	9.32		
	9/11	1930	0.86	1.53	2.59	4.52	6.49	8.53	0.216	8.4
			1.49	2.53	3.52	5.18	6.23	7.17		
	9/12	0140	1.02	2.21	3.12	4.91	7.05	9.38	0.201	10.8
			2.25	4.03	5.02	6.49	7.97	9.35		
	9/12	0750	0.96	1.96	2.88	4.83	6.81	9.12	0.218	11.1
			2.53	3.96	5.03	6.98	8.66	9.67		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/l						K_1, per Day	Ultimate First Stage BOD, mg/l
475.1	9/12	1535	0.67	1.67	2.58	4.60	6.52	8.83	0.233	9.0
			1.48	3.08	4.09	5.63	6.91	8.00		
	9/12	2315	1.03	1.95	2.94	4.89	6.95	8.99	0.226	8.7
			2.04	3.30	4.13	5.62	6.80	7.76		
476.2	9/11	0825	0.96	1.83	2.92	4.82	6.79	8.83	0.241	9.7
			2.29	3.66	4.84	6.52	7.51	8.89		
	9/11	1615	0.88	1.54	2.62	4.53	6.50	8.54	0.207	8.3
			1.52	2.35	3.35	4.91	5.97	7.05		
	9/12	0315	1.02	2.21	3.12	4.91	7.06	9.38	0.151	10.8
			1.79	3.07	4.09	5.39	7.31	8.17		
	9/12	0900	0.96	1.96	2.88	4.83	6.81	9.02	0.254	9.6
			2.35	4.01	4.84	6.64	7.72	8.90		
	9/12	1615	0.66	1.66	2.57	4.59	6.51	8.82	0.186	9.5
			1.23	2.67	3.72	5.15	6.52	7.78		
	9/13	0010	0.98	1.90	2.89	4.83	6.90	8.94	0.197	8.5
			1.73	2.81	3.64	4.99	6.30	7.16		
477.5	9/11	0900	0.98	1.85	2.79	4.84	6.89	8.81	0.218	8.5
			1.76	2.84	3.96	5.49	6.48	7.44		
	9/11	1655	0.89	1.55	2.64	4.54	6.51	8.55	0.211	7.6
			1.47	2.23	3.37	4.56	5.30	6.66		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/l						K_1, per Day	Ultimate First Stage BOD, mg/l
477.5	9/12	0400	1.02	2.21	3.12	4.91	7.06	9.38	0.164	8.1
			1.32	2.33	3.14	4.30	5.47	6.40		
	9/12	0940	0.96	1.96	2.88	4.90	6.81	9.12	0.181	9.8
			1.84	3.01	3.86	5.59	6.88	7.97		
	9/12	1650	0.64	1.64	2.55	4.57	6.49	8.80	0.193	9.2
			1.23	2.72	3.45	5.34	6.50	7.64		
	9/13	0030	1.01	1.93	2.92	4.86	6.93	8.93	0.127	9.5
			1.26	2.16	2.81	4.40	5.58	6.47		
479.1	9/11	0940	1.00	1.88	2.96	4.86	6.83	8.88	0.195	8.4
			1.55	2.60	3.73	5.05	6.00	7.01		
	9/11	1730	0.90	1.56	2.65	4.55	6.52	8.56	0.105	10.5
			1.13	1.71	2.62	3.88	5.06	6.39		
	9/12	0445	1.02	2.21	3.10	4.91	7.06	9.38	0.135	8.5
			1.22	2.20	2.85	4.13	5.10	6.15		
	9/12	1025	0.96	1.96	2.88	4.90	6.81	9.12	0.142	9.4
			1.34	2.37	3.20	4.44	5.30	6.87		
	9/12	1725	0.62	1.62	2.54	4.56	6.48	8.79	0.120	9.6
			0.82	1.79	2.62	3.92	5.08	6.37		
	9/13	0050	0.96	1.88	2.86	4.81	6.88	8.92	0.122	9.4
			1.11	2.06	2.78	4.16	5.17	6.38		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
474.60	10/9	0915	0.87	1.87	2.92	4.75	6.67	8.65	0.207	6.7
			1.61	2.41	3.07	3.84	4.68	6.04		
	10/9	1510	0.96	1.56	2.60	4.44	6.35	8.33	0.185	9.0
			1.86	2.62	3.42	4.64	5.83	7.43		
	10/9	2245	1.19	2.54	3.23	5.10	6.98	9.08	0.128	12.2
			2.06	3.48	4.17	5.62	6.97	8.60		
	10/10	0705	0.83	1.98	2.87	4.75	6.62	8.23	0.154	9.5
			1.59	2.77	3.42	4.48	6.00	7.12		
	10/10	1500	0.46	1.60	2.50	4.38	6.25	8.35	0.157	10.4
			0.87	2.74	3.50	4.86	6.17	7.92		
	10/10	2205	1.13	2.02	2.85	4.77	6.75	8.90	0.196	7.5
			1.82	2.64	3.29	4.20	5.19	6.52		
475.1	10/9	0945	0.87	1.87	2.92	4.75	6.67	8.65	0.286	5.3
			1.61	2.34	2.79	3.62	4.41	5.04		
	10/9	1535	0.96	1.56	2.60	4.44	6.35	8.33	0.208	7.5
			1.68	2.33	3.03	4.36	5.09	6.48		
	10/9	2315	1.19	2.54	3.23	5.10	6.98	9.08	0.090	13.0
			1.70	2.82	3.40	4.34	6.05	7.44		
	10/10	0740	0.83	1.98	2.87	4.75	6.62	8.23	0.174	7.6
			1.44	2.46	3.13	3.76	4.90	6.20		

Station Mileage	Collection of Sample Date	Collection of Sample Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
475.1	10/10	1525	0.46	1.60	2.50	4.38	6.25	8.35	0.163	8.5
			0.82	2.23	3.02	4.07	5.16	6.61		
	10/10	2250	1.13	2.02	2.85	4.77	6.75	8.90	0.168	9.0
			1.79	2.86	3.63	4.54	5.67	7.37		
476.2	10/9	1030	0.87	1.87	2.92	4.75	6.67	8.65	0.274	5.7
			1.57	2.34	3.10	3.93	4.69	5.38		
	10/9	1605	0.96	1.56	2.60	4.44	6.35	8.33	0.119	9.1
			1.39	1.85	2.45	3.51	4.36	6.07		
	10/9	2350	1.19	2.54	3.23	5.10	6.98	9.08	0.154	8.8
			1.74	2.98	3.55	4.52	5.60	6.88		
	10/10	0855	0.82	1.98	2.87	4.75	6.62	8.23	0.260	5.8
			1.34	2.48	2.66	4.36	4.78	5.13		
	10/10	1600	0.54	1.69	2.58	3.42	6.33	8.44	0.119	7.6
			0.57	1.54	1.96	2.48	3.95	4.87		
	10/10	2205	1.13	2.02	2.85	4.77	6.75	8.90	0.182	8.6
			1.94	2.89	3.62	4.63	5.64	7.34		
477.6	10/9	1100	0.87	1.87	2.92	4.75	6.67	8.65	0.252	6.5
			1.56	2.68	3.50	4.20	4.89	6.25		
	10/9	1625	0.96	1.56	2.60	4.44	6.35	8.33	0.165	6.8
			1.37	1.74	2.34	3.36	4.12	5.36		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
477.6	10/10	0925	0.82	1.98	2.87	4.75	6.62	8.23	0.117	10.2
			1.28	2.40	3.00	4.11	5.01	6.75		
	10/10	1625	0.54	1.69	2.58	3.42	6.33	8.44	0.130	6.3
			0.60	1.45	1.83	2.15	3.32	4.36		
	10/10	2305	1.13	2.02	2.85	4.77	6.75	8.90	0.107	11.3
			1.73	2.52	3.03	4.13	5.49	7.26		
479.1	10/9	1125	0.87	1.87	2.29	4.75	6.67	8.65	0.313	6.0
			1.58	2.52	3.36	4.32	5.16	5.80		
	10/9	1650	0.96	1.56	2.60	4.44	6.35	8.33	0.146	7.3
			1.28	1.60	2.16	3.46	4.26	5.29		
	10/10	0045	1.19	2.54	3.23	5.10	6.93	9.08	0.097	9.8
			1.40	2.22	2.66	3.62	4.48	5.93		
	10/10	1000	0.83	1.98	2.87	4.75	6.62	8.23	0.165	8.1
			1.28	2.46	3.21	4.08	4.99	6.41		
	10/10	1645	0.54	1.69	2.58	3.42	6.33	8.44	0.152	5.8
			0.59	1.58	1.90	2.36	3.22	4.49		
	10/10	2355	1.13	2.02	2.85	4.77	6.75	8.90	0.125	8.5
			1.44	2.16	2.66	3.54	4.36	6.06		
474.6	10/22	1240	0.88	1.88	2.75	4.88	6.69	8.62	0.093	21.1
			2.10	3.42	4.72	6.75	9.63	11.58		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
474.6	10/22	2010	0.90	1.81	2.40	4.52	6.33	9.26	0.122	26.6
			4.09	6.19	7.20	10.43	12.94	18.87		
	10/23	0340	2.04	3.23	5.00	6.96	8.91		0.084	21.8
			4.05	5.43	6.95	9.07	11.91			
	10/23	1140	1.04	1.79	2.98	4.75	6.71		0.191	13.9
			2.60	4.61	5.88	7.60	10.44			
	10/23	1925	0.73	1.35	2.54	4.31	6.27	8.22	0.173	18.2
			2.97	4.88	6.72	8.71	10.87	14.86		
	10/24	0330	1.04	2.23	3.17	4.98	6.92		0.121	25.7
			3.42	6.32	7.74	11.57	14.67			
	10/24	1120	0.94	1.88	2.81	4.62	6.56	8.92	0.048	32.4
			1.54	2.60	3.57	6.50	9.10	10.90		
476.2	10/22	1330	0.88	1.88	2.75	4.88	6.69	8.62	0.051	24.8
			1.50	2.65	3.51	5.07	6.66	9.23		
	10/23	0425	2.04	3.23	5.00	6.96	8.91		0.116	25.2
			6.03	7.72	11.18	13.14	16.81			
	10/23	1400	1.01	1.76	2.95	4.72	7.68		0.204	11.2
			2.30	3.59	4.98	6.65	9.02			
	10/23	2010	0.78	1.35	2.54	4.31	6.27	8.22	0.143	17.9
			2.35	3.67	5.82	7.61	9.99	12.95		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
476.2	10/24	0905	1.04	2.23	3.17	4.98	6.92		0.084	29.7
			2.76	5.22	6.42	10.23	13.02			
	10/24	1150	0.94	1.88	2.81	4.62	6.56	8.92	0.053	25.4
			1.18	2.26	3.22	5.44	8.15	9.25		
	10/24	2115	0.59	1.53	2.47	4.28	6.22	9.53	0.142	13.4
			1.38	2.76	3.78	5.94	7.97	9.91		
477.5	10/22	1405	0.88	1.88	2.75	4.88	6.69	8.62	0.065	19.0
			1.39	2.56	3.31	4.79	6.42	8.50		
	10/23	0455	2.04	3.23	5.00	6.96	8.91		0.096	17.7
			3.55	5.03	6.35	8.19	10.56			
	10/23	1345	0.94	1.69	3.[illegible]	4.65	6.60		0.226	10.2
			2.01	3.75	5.32	6.74	8.14			
	10/23	2040	0.73	1.35	2.54	4.31	6.27	8.22	0.082	20.6
			1.63	2.65	4.16	5.92	7.42	10.66		
	10/24	2150	0.59	1.53	2.47	4.28	6.22	9.53	0.061	20.7
			1.06	2.24	2.70	4.38	6.83	9.11		
479.1	10/23	1420	0.94	1.69	2.[illegible]	4.65	6.60		0.314	8.1
			2.39	3.26	4.66	6.06	7.16			
	10/23	2100	0.73	1.35	2.54	4.31	6.27	8.22	0.036	40.4
			1.20	1.86	3.29	5.55	6.85	9.85		

Station Mileage	Collection of Sample Date	Time	Elapsed Time, Days Observed Biochemical Oxygen Demand, mg/1						K_1, per Day	Ultimate First Stage BOD, mg/1
481.45	10/23	1500	0.94	1.69	2.88	4.65	6.60		0.178	8.3
			1.34	2.30	3.32	4.42	5.86			
	10/23	2140	0.73	1.35	2.54	4.31	6.27	8.22	0.159	11.3
			1.32	2.58	3.98	5.33	6.55	8.67		
	10/24	0530	1.04	2.23	3.17	4.98	6.92		0.295	10.9
			2.96	5.20	6.38	8.80	9.29			

APPENDIX E

HYDRAULIC CHARACTERISTICS OF REACHES IN TENNESSEE VALLEY RIVERS USED IN MULTIPLE CORRELATION ANALYSIS

*Experiment Number	River	K_2 (per Day) No. of Observations	K_2 (per Day) Geometric Mean Value	K_2 (per Day) Arithmetic Mean Value (Z'_o)	Mean Velocity V, fps	Mean Depth H, feet	Predicted Value (Z_o) (per Day)	$\frac{Z'_o - Z_o}{Z_o} \times 100$
1	Clinch	19	2.272	2.920	3.07	3.27	2.177	34.13
2	Clinch	19	1.440	1.449	3.69	5.09	1.213	19.46
3	Clinch	25	0.981	1.061	2.10	4.42	0.917	15.70
4	Clinch	29	0.496	0.550	2.68	6.14	0.656	-16.16
5	Clinch	29	0.743	0.842	2.78	5.66	0.799	5.38
6	Clinch	30	1.129	1.170	2.64	7.17	0.496	125.80
7	Holston	26	0.281	0.315	2.92	11.41	0.247	27.53
8	Holston	30	3.361	3.422	2.47	2.12	3.729	- 8.23
9	Holston	16	2.794	2.819	3.44	2.93	2.916	- 3.33
10	Holston	5	1.568	1.574	4.65	4.54	1.825	-13.75
11	Holston	31	0.455	0.505	2.94	9.50	0.339	47.49
12	Holston	26	0.389	0.420	2.51	6.29	0.592	-29.05

*Experiment Number	River	K_2 (per Day) No. of Observations	K_2 (per Day) Geometric Mean Value	K_2 (per Day) Arithmetic Mean Value (Z'_o)	Mean Velocity V, fps	Mean Depth H, feet	Predicted Value (Z_o) (per Day)	$\frac{Z'_o - Z_o}{Z_o} \times 100$
13	Holston	27	0.270	0.300	3.15	7.52	0.539	-44.34
14	Holston	26	0.550	0.660	3.30	7.07	0.625	5.60
15	Holston	18	0.544	0.559	3.11	5.44	0.925	-39.57
16	Holston	7	0.604	0.670	4.28	3.06	0.635	5.51
17	Holston	20	1.251	1.309	2.73	3.88	1.459	-10.2[illegible]
18	French Broad	8	0.273	0.284	2.41	9.28	0.294	- 3.40
19	French Broad	8	0.225	0.261	3.06	10.19	0.312	-16.35
20	French Broad	8	1.881	1.896	2.40	3.29	1.716	10.49
21	French Broad	8	0.842	0.870	3.46	4.74	1.291	-32.61
22	French Broad	6	0.883	0.934	4.02	5.72	0.930	0.43
23	French Broad	7	0.915	0.983	4.52	6.95	0.860	14.30
24	French Broad	7	0.995	1.006	1.85	4.29	0.858	17.25

*Experiment Number	River	K_2 (per Day) No. of Observations	K_2 (per Day) Geometric Mean Value	K_2 (per Day) Arithmetic Mean Value (Z'_o)	Mean Velocity V, fps	Mean Depth H, feet	Predicted Value (Z_o) (per Day)	$\frac{Z'_o - Z_o}{Z_o} \times 100$
25	French Broad	8	0.547	0.557	2.75	6.01	0.696	-19.97
26	French Broad	8	0.881	0.903	3.23	7.16	0.599	50.75
27	French Broad	7	0.252	0.312	3.71	8.49	0.509	-38.70
28	Watauga	16	5.558	5.858	5.00	3.42	3.164	85.15
29	Hiwassee	16	1.712	1.812	3.05	3.02	2.478	-26.88
30	Hiwassee	19	3.222	3.265	3.51	2.83	3.152	3.59

*See Table 1 of Churchill, et al. (1962), for experiment number identification.